MATHEMATICS FRAMEWORK

for the 2026 NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

NATIONAL ASSESSMENT GOVERNING BOARD
U.S. Department of Education

WHAT IS NAEP?

The National Assessment of Educational Progress (NAEP) is a continuing and nationally representative measure of trends in academic achievement of U.S. elementary and secondary students in various subjects. For nearly four decades, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, U.S. history, civics, geography, and other subjects. By collecting and reporting information on student performance at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education.

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CHAPTER 1

OVERVIEW

The National Assessment of Educational Progress (NAEP) has measured student achievement nationally since 1973, and state-by-state since the early 1990s, providing the nation with a snapshot of what students in this country know and can do in mathematics. Starting in 2002, urban school districts that meet certain selection criteria could volunteer to participate in the Trial Urban District NAEP Assessment.

Frameworks are designed to inform NAEP assessment development; they describe the subject matter to be assessed and the assessment questions to be asked, as well as the assessment's design and administration. The major purpose for this NAEP Mathematics Framework is to identify what mathematics should be measured on NAEP at grades 4, 8, and 12 beginning in 2026. The most recent updates of the NAEP Mathematics Framework were completed in 2001 for grades 4 and 8, and in 2006 for grades 4, 8, and 12 and were reflected in the 2005 and 2009 and succeeding NAEP Mathematics Assessments, respectively.

This framework offers guidance for how developments in educational research, policy, and practice over the past two decades should be reflected in the NAEP Mathematics Assessment. This updated framework is based on a visioning and development process that engaged mathematics educators, curriculum experts, researchers, assessment experts, teachers, and other leading educators. A major goal in the process was to ensure that NAEP is designed and implemented in ways that allow *all* students to show their best work in terms of what they know and can do mathematically. This means ensuring maximum accessibility to different groups of students who live and learn in a wide range of contexts, including urban, rural, and suburban; who have a wide spectrum of experiences, backgrounds, and needs; and who represent a wide range of communities of different ethnic, cultural, and linguistic strengths and in- and out-of-school experiences. This framework update is based on a growing awareness, in research and practice, of the significance of these differences in teaching and learning.

There are several important audiences for this framework. Primary among these are educators in schools, policymakers, students and their families, and the general public. In addition, this framework and the accompanying NAEP Mathematics *Assessment and Item Specifications* document are for the National Center for Education Statistics (NCES) and its contractors, critical NAEP partners, who will use both documents to develop the 2026 NAEP Mathematics Assessment.

Background on NAEP

There are two distinct components to the NAEP Mathematics Assessment, which differ in purpose. The NAEP Long-Term Trend assessment has measured trends in achievement among 9-, 13-, and 17-year-old students nationally since 1973, and the assessment's content has been essentially unchanged ever since. The second assessment, referred to as "main NAEP," is adjusted over time to reflect shifts in research, policy, and practice. The content and format of the main NAEP Mathematics Assessment are the focus of this framework.

The main NAEP Mathematics Assessment is administered at the national, state, and selected urban district levels every two years, by Congressional mandate. In mathematics, NAEP results are reported on student achievement in grades 4, 8, and 12 at the national level, and for grades 4 and 8 at the state level and for large urban districts that volunteer to participate.

Taken together, the NAEP assessments provide a rich and broad picture of patterns in U.S. student mathematics achievement. National and state level results are reported in terms of scale scores, achievement levels, and percentiles. These reports provide comprehensive information about what U.S. students know and can do in mathematics. In addition, NAEP provides comparative subgroup data according to gender, race/ethnicity, socioeconomic status, and geographic region; describes trends in performance over time; and reports on relationships between student achievement and certain contextual variables.

The main NAEP assessment is administered to a nationally representative sample of students and reports on student achievement in the aggregate. The assessment is not designed to measure the performance of any individual student or school. To obtain reliable estimates across the population that is assessed, a large pool of assessment items is developed. Subsets of items are administered to each student in the sample. Student results on the main NAEP assessments are reported for three achievement levels established and defined by the National Assessment Governing Board (Governing Board), which oversees NAEP:

- *NAEP Basic* denotes partial mastery of prerequisite knowledge and skills that are fundamental for performance at the *NAEP Proficient* level.
- *NAEP Proficient* represents solid academic performance for each NAEP assessment. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
- *NAEP Advanced* signifies superior performance beyond *NAEP Proficient*.

These policy definitions can be found in the Governing Board's *Developing Student Achievement Levels for the National Assessment of Educational Progress Policy Statement* (2018b). Descriptions and examples of student performance at these levels of achievement at grades 4, 8, and 12 for this framework are provided in Appendices A and B, respectively. Chapter 5 includes further discussion of the achievement levels.

This document describes an assessment framework, not a curriculum framework. It lays out the basic design of the assessment by describing the mathematics content and mathematical practices that should be assessed and the types of questions that should be included. It also describes how various assessment design factors should be balanced across the assessment. In broad terms, this framework attempts to answer the question: What mathematics knowledge, skills, and practices are to be assessed on NAEP at grades 4, 8, and 12? It does not cover all relevant content for each grade level; some concepts, practices, and activities in school mathematics are not suitable to be assessed on NAEP, although they may well be important components of a school curriculum. For example, the practice of extended investigation would not be possible in the NAEP assessment, although it would be quite reasonable for teachers to have multi-day investigations of some important mathematical ideas. This document also does not attempt to answer the question: How should mathematics be taught?

The Visioning and Development Process

The process for updating the mathematics assessment framework consisted of a review by experts in mathematics education research, policy, and practice representing key stakeholder groups. This process—which is described in the Governing Board's *Framework Development Policy Statement* (2018c)—involved visioning for the update, and then development.

The Visioning Panel was tasked with formulating "high-level guidance about the state of the field to inform the process." The specific charge stated:

> The Visioning and Development Panels will recommend to the Governing Board how best to balance necessary changes in the NAEP Mathematics Framework at grades 4, 8, and 12 with the Governing Board's desire for stable reporting of student achievement trends and assessment of a broad range of knowledge and skills, so as to maximize the value of NAEP to the nation; and the Panels are also tasked with considering opportunities to extend the depth of measurement and reporting given the affordances of digitally based assessment.

The 30-person Visioning Panel met in November 2018 to determine principles, goals, and policies to guide the NAEP Mathematics Framework update. During this meeting, the Visioning Panel learned about NAEP, the framework update process, and available NCES resources. Using this information, panelists identified and discussed issues related to developments in mathematics education research, policy, and practice that should inform the design of the assessment framework. The Visioning Panel then developed guidelines for recommended updates. The guidelines were clustered in three domains: mathematics, assessment design and technology, and opportunities to learn. These are summarized in Appendix C.

The full set of guidelines was passed on to the Development Panel, fifteen Visioning Panelists who were tasked with developing drafts of updated project documents and engaging in deliberations about how issues outlined in the guidelines should be reflected in the framework. The three documents include: a recommended framework, assessment and item specifications, and recommendations for contextual variables that relate to the subject being assessed. The Development Panel convened four 2-day meetings to prepare these three documents, as well as webinars to prepare for and review progress. In between and after meetings, the Development Panel drafted and revised documents. The updates included responding to the guidelines set by the Visioning Panel. In addition, all updates were made in congruence with Governing Board policies. The Development Panel drew on a wide range of policy and research documents to inform its deliberations, including a review of the 2017 NAEP Mathematics Framework that was commissioned by the Governing Board (2018a). All of the sources that informed the Panel's work are listed in the references.

Complementary to the Visioning and Development Panels, a Technical Advisory Committee (TAC) of eight recognized measurement experts advised the panels about technical issues. The TAC met six times, and representatives attended the panel meetings. The TAC made recommendations concerning content and cognitive dimensions in the framework, as well as item and assessment design.

Overview of Assessment Design and Framework Chapters

The proposed design for the 2026 assessment aims to provide a fair and valid measure of how well all students have achieved the depth and breadth of the mathematics content and practice articulated by this framework. To do this, the design:

- incorporates a mix of traditional and innovative item types that reflect recent research on the science of learning, to capture both the process and outcomes of student learning, and emphasizes authentic applications of mathematics knowledge and skill;
- capitalizes on the use of technology to assure accessibility, promote engagement for all students, and explore new options for task design and scoring, including the use of multimedia;
- encourages continuing prototyping and research to capitalize on the capacities of current and emerging technology to assess students at deeper levels, while still ensuring validity and fairness of scores; and
- recognizes the potential of technology and new task designs while also acknowledging limitations and potential negative unintended consequences. The design plan is a careful balance to promote more valid assessment of mathematics content and practices without compromising fairness or reliability (e.g., fairness for students who have less access to technology, scenarios that avoid construct-irrelevant barriers of language, and innovative task types that reduce the number of items).

This framework consists of five chapters and several appendices. Chapter 2 describes the content areas: Number Properties and Operations (including computation and understanding of number concepts); Measurement (including use of instruments and concepts of area and volume); Geometry (including spatial reasoning and applying geometric properties); Data Analysis, Statistics, and Probability (including graphical displays and statistical measures); and Algebra (including representations and relationships). Each content area is broken into subtopics (e.g., for Number Properties and Operations, these are number sense, estimation, number operations, ratios and proportional reasoning, and properties of number and operations) identifying what should be measured on NAEP at grades 4, 8, and 12.

Chapter 3 describes the NAEP Mathematical Practices that play a role in measuring student knowledge and skills in mathematics. These are Representing, Abstracting and Generalizing, Justifying and Proving, Mathematical Modeling, and Collaborative Mathematics. The chapter argues that content and practices are interwoven and interdependent: one cannot demonstrate mathematics achievement without knowing content and being able to think mathematically. Chapter 3 also offers example items across grades 4, 8, and 12 that illustrate how NAEP Mathematical Practices can be assessed with particular content.

Chapter 4 focuses on issues of technology and accessibility, assessment design, and item format. The chapter argues for the need to ground the NAEP Mathematics Assessment in tasks in familiar contexts to foster student engagement. By expanding item types and thoughtfully using technology, the NAEP Mathematics Assessment can provide greater access to all students, diversify the ways in which student achievement can be recognized and measured, and more robustly assess both what students know and what they can do. This will involve expanding the assessment to include scenario-based tasks (which involve clusters of related items within one task) along with continued use of existing discrete NAEP items that capture student understanding of content and mathematical practices. As the technology of assessment evolves, alternative formats might also be considered.

Chapter 5 addresses how NAEP results are reported. The chapter describes the three NAEP achievement levels and the development of the mathematics achievement level descriptions (see Appendix A). The chapter builds on an expansive conception of "opportunity to learn" as called for by the Visioning Panel Guidelines (see Appendix C). The chapter also discusses how research on student diversity and schooling informs mathematics-specific contextual variables.

Opportunity to Learn and an Expansive Understanding of Contextual Variables

What students learn is inseparable from the conditions of their learning and broader social aspects of mathematics learning. Hence, interpreting differences in what students can do on NAEP requires an understanding of the range of factors that affect student learning. In particular, this framework articulates an expansive conception of opportunities to learn, informed by educational research on students and their in- and out-of-school learning and experiences, as well as research on the variations in human, material, and social resources that shape what students have an opportunity to learn about mathematics in the U.S. (e.g., Cohen, Raudenbush, & Ball, 2003; Tatto et al., 2012).

Opportunity to learn is generally understood to refer to inputs and processes that shape student achievement, including the school conditions, curriculum, instruction, and resources to which students have access. When opportunity to learn was first used as a concept, Carroll (1963, 1989) emphasized the time allowed for learning. For the past 50 years, the concept of opportunity to learn has continued to evolve, as have efforts to measure in-school opportunities to learn, with the majority of scholars focusing on the classroom as the unit of analysis and instruction as central. Research, for example, has documented the negative effects on achievement of policies and practices that are often found in schools serving children who live in poverty or have special needs, including an inadequate supply of mathematics teachers with strong knowledge and skills, a tendency to offer few advanced mathematics courses, and a common practice of tracking these students disproportionately into low-level courses that restrict their learning opportunities (e.g., Husén, 1967; Tan & Kastberg, 2017), all of which can be understood as instructional resources that shape what students learn.

Important to note is the sociopolitical turn that has taken place in research on school mathematics (Gutiérrez, 2013), which positions mathematics as a "dynamic, political, historical, relational, and cultural subject" (TODOS & NCSM, 2016, p. 3) in which identity and power both play central roles. This turn has led scholars and educators to explore how school mathematics marginalizes and alienates students who do not see connections to their own lives and experiences. It raises questions about how school mathematics might be reformed to engage all students and their communities. This includes students with disabilities who are often relegated to classrooms where learning differences are conceptualized as a deficit rather than a potential strength and where the focus is on procedural approaches rather than leveraging students' own particular strategies to engage in mathematical reasoning and sense making (e.g., Lambert, Tan, Hunt, & Candella, 2018).

Another noteworthy development in mathematics education research is acknowledgment that students themselves are a resource in learning, including their interests, abilities, and in- and out-of-school experiences. Research, for example, suggests that students' experiences out-of-school can be directly relevant to the ways they think mathematically and use mathematics (e.g., Martin, 2000; Nasir & Hand, 2008). Some scholars refer to this as students' "funds of knowledge," defined as the skills, knowledge, habits of mind, practices, and experiences acquired through historical and cultural interactions of an individual in their community, family life, and culture through everyday living as well as in school (e.g., Aguirre et al., 2013; Civil, 2016; de Freitas & Sinclair, 2016; González, Moll, & Amanti, 2005; Moll, Amanti, Neff, & González, 1992). Students' funds of knowledge include what has often been referred to as students' prior knowledge, but expands that idea to include cultural, linguistic, and social traditions that characterize students' lives out of school. While these funds of knowledge might differ from those of the teacher or the traditional curriculum, the broad experiences of students can be used to make powerful connections that enable learning and can be understood as an additional resource in instruction and assessment. Therefore, this framework's conception of opportunity to learn includes students' experiences, out-of-school learning, and funds of knowledge as an instructional resource.

Relevant opportunity to learn indicators have been clustered in various ways (e.g., Abedi & Herman, 2010; Elliott & Bartlett, 2016; Herman, Klein, & Abedi, 2000; Husén, 1967; Schmidt, Burroughs, Zoido, & Houang, 2015; Wang, 1998). These can be grouped into five strands: time, content and practices, instructional strategies, teacher factors, and instruction-relevant resources. Examples of indicators that have been used in research are provided in Exhibit 1.1.

To support audiences in interpreting NAEP results, information about contextual variables is collected through student, teacher, and administrator surveys. The framework development process drew broadly on the literature to create an ambitious conception of opportunity to learn as the basis for recommendations about mathematics-specific contextual variables on NAEP surveys. As is the case with mathematics content, it is neither possible nor appropriate to measure all potentially relevant contextual variables on NAEP. For example, questions that ask students about their home or out-of-school experiences can be experienced as intrusive. Priorities for the selection of mathematics-relevant variables are described in Chapter 5.

Exhibit 1.1. Opportunity to Learn Strands

Strand	Example Indicators
Time	time scheduled for instruction proportion of allocated time used for instruction time students are engaged in learning time students are experiencing a high success rate of learning
Content and Practices	content and practices exposure content and practices emphasis content and practices coverage
Instructional Strategies	instructional approaches (e.g., strategies that facilitate student thinking and understanding, instruction that promotes student engagement) classroom climate instructional group size
Teacher Factors	teacher preparation and professional development teacher knowledge, including mathematical knowledge for teaching teaching experience teacher attitudes about themselves, students, learning, and mathematics
Instruction-Relevant Resources	material resources (e.g., textbooks, manipulatives) school policies (e.g., tracking) school community and climate; school and instructional leadership students' experiences, out-of-school learning, and funds of knowledge student access to technological tools

Major Changes in This Framework

This update of the NAEP Mathematics Framework reflects several major changes. The changes are summarized in the following sections and elaborated in Exhibit 1.2 at the end of this chapter.

Mathematics Content

Chapter 2 presents an updated set of content objectives for the 2026 NAEP Mathematics Assessment at grades 4, 8, and 12. The updates reflect the last decade of changes in state standards for mathematics curriculum, instruction, and assessment. State standards shape what students have had an opportunity to learn by the time they take a NAEP assessment. To ensure the updates reflect current state-level emphases for mathematics content, this framework incorporates findings from several reports that compared NAEP and state standards (e.g., Achieve, 2016; Johnston, Stephens, & Ratway, 2018), as well as reports on the mathematics content taught in leading countries around the world (e.g., as assessed in the Trends in International Mathematics and Science Study [TIMSS] [NCES, 2019] and the Programme for International Student Assessment [PISA] [OECD, 2019]). Because this framework has been written for an assessment in 2026 and beyond, it is also informed by national policy that foreshadows likely changes in state policy (e.g., Bargagliotti et al., 2020; Garfunkel & Montgomery, 2019).

Mathematical Literacy

In every state, all high school graduates are required to study mathematics whether or not their future plans involve college or a field in which high school mathematics is heavily involved. The purpose of this universal practice is to ensure that the U.S. citizenry is mathematically literate. Recent policy developments have included attention to mathematical literacy, for example, in mathematical modeling of real-world problems and interpreting reports of data.

Mathematical literacy is the ability to apply mathematical concepts to everyday situations. It has been recognized worldwide as important. In 2015, the PISA assessments, given to 15-year-olds every three years, were conducted in 70 countries, more countries than any other mathematics assessment (OECD, 2018). The PISA assessments emphasize mathematical literacy and define it as the application of numerical, spatial, or symbolic mathematical information to situations in a person's life as a consumer, employee, or citizen. The definition for this framework is based on the PISA definition, given its extensive, worldwide use and given the availability of assessment items that have been created following that definition:

> *Mathematical literacy is the application of numerical, spatial, or symbolic mathematical information to situations in a person's life as a community member, citizen, worker, or consumer.*

A large body of experiences can be viewed as requiring mathematical literacy, including fluency in the broad range of mathematics of personal finances; understanding statistical information and displays found in print and visual media; and household tasks such as cooking, cleaning, and furnishing that require a variety of measurements. For example, mathematical literacy affects how one critically evaluates reports on environmental issues, estimates how many bricks are needed to build a walkway, or compares interest rates for a loan. Mathematical literacy is part of the everyday experiences that occur in community, civic, professional, and personal contexts of adults in the United States, regardless of career.

At grades 4 and 8, instances of mathematical literacy are found in the standard content taught in schools, have been in previous NAEP frameworks, and remain in the objectives enumerated here. At grade 12, historically, instances of mathematical literacy have been given less attention. In this framework, throughout grade 12, objectives that provide opportunities for assessment of mathematical literacy are identified by the number/hashtag sign (#). See Chapter 2 for more on the issue of mathematical literacy.

NAEP Mathematical Practices

Since the late 1980s, there have been ongoing efforts to more clearly specify mathematical processes like "higher-order thinking" or "mathematical reasoning." Current conceptions of mathematical knowledge and skill have shifted to specify mathematical practices and processes. At the turn of the 21st century, in *Adding It Up*, the National Research Council (NRC, 2001) enumerated five strands of mathematical proficiency, including:

- *conceptual understanding*: comprehension of mathematical concepts, operations, and relations;
- *procedural fluency*: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

- *strategic competence*: ability to formulate, represent, and solve mathematical problems;
- *adaptive reasoning*: capacity for logical thought, reflection, explanation, and justification; and
- *productive disposition*: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

For decades, the National Council of Teachers of Mathematics (NCTM) has discussed five mathematical processes standards: problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000). Processes like these have been central to NAEP frameworks for the last 20 years and state standards have reiterated the important role of practices. The language of "practice" has become increasingly popular, establishing a foothold through various state standards, as well as in discussions of teaching with and through practices (NCTM, 2014). This framework provides the following definition:

> *NAEP Mathematical Practices are the routines, norms, and processes needed to do the work of mathematics.*

Based on the current state of the field, this framework identifies five NAEP Mathematical Practices for the NAEP Mathematics Assessment:

NAEP Mathematical Practice 1: Representing
NAEP Mathematical Practice 2: Abstracting and Generalizing
NAEP Mathematical Practice 3: Justifying and Proving
NAEP Mathematical Practice 4: Mathematical Modeling
NAEP Mathematical Practice 5: Collaborative Mathematics

These mathematical practices are described in depth in Chapter 3. Note that these mathematical practices are not instructional practices used by teachers. They are the actions necessary to do mathematics. This list of NAEP Mathematical Practices also does not endorse one particular view of mathematical practices (an issue further discussed in Chapter 3).

Item Formats and Technology in Assessment

A fourth major change involves item formats and the role of technology in assessment. As noted previously and as further explained in Chapter 4, technological innovation is relevant to NAEP because it allows for more authentic assessments and for a broader range of accommodations to meet students' needs.

Since 1992, the NAEP Mathematics Assessment has used two types of items (questions): multiple choice and constructed response. In 2017, the NAEP assessment began to include these item formats in a digital platform as part of the NAEP transition to digitally based assessment. The transition to digital administration provided opportunities to expand the range of formats used for items.

In advancing the expansion of item types and formats, three themes emerged. One theme concerns how research on students' knowledge and experience can be used to design assessments that capture their capacity to do mathematics. This includes the use of interactive, multimedia scenario-based tasks to assess what students know and can do. Scenario-based tasks currently exist in other NAEP assessments, including NAEP Science and NAEP Technology and Engineering Literacy.

By expanding item formats, to include scenario-based tasks (and new item formats that emerge in the future) and to thoughtfully use technology, the aim is to provide greater access to all students, as well as to diversify the ways in which student achievement can be recognized and measured. Note that technological innovation is not just limited to enhancing assessment accommodations. Technology is a part of every student's life and learning, and mathematical thinking can be enhanced by its judicious use.

A second theme concerns the use of technology to enable assessment of the NAEP Mathematical Practices, including an expanded range of response types leveraging object-based and discourse responses within a scenario-based task. Less often noted but equally important is a third theme concerning the intended or unintended negative consequences of technology, which include inequitable access to technologies. That is, while technology may have the potential to increase access and opportunities to demonstrate learning, students unfamiliar with technologies used in the assessment could be at a disadvantage. With the introduction of scenario-based tasks it is critical to ensure that students have ample time to understand how to engage with assessment items along with opportunities to experience the task type.

Changes from the 2009–2017 Framework

Exhibit 1.2 compares this framework for the 2026 NAEP Mathematics Assessment and those used for the 2009–2017 NAEP Mathematics Assessments. The focus here is on major changes. Many of the points summarized below are expanded in Chapters 2, 3, and 4. Justifications for these changes are briefly described below, with more details in the relevant chapters.

Exhibit 1.2. Summary of Changes in the 2026 NAEP Mathematics Framework

Topic	Change	Rationale
Mathematics Content	Many objectives were edited to increase clarity and specificity.	Objectives and balance of topics were updated to reflect shifts in expectations evident from reviews of state and national standards, policy documents from leading professional organizations, and expectations for mathematical literacy on U.S. and international assessments. For more details on changes, see Chapter 2.
	The objectives in the mathematical reasoning subtopics have been removed. This subtopic was introduced in 2009 for Number Properties and Operations; Geometry; Data Analysis, Statistics, and Probability; and Algebra.	With the introduction of the NAEP Mathematical Practices (see Chapter 3), mathematical reasoning was no longer needed as a subtopic. To preserve attention to the content that was uniquely present in some of the mathematical reasoning objectives, objectives in other subtopics were revised. For more details on changes, see Chapter 2.

Exhibit 1.2. Summary of Changes (continued)

Topic	Change	Rationale
Mathematics Content (continued)	Distribution of items for grade 12 remains the same. The proportion of Data Analysis, Statistics, and Probability items has increased for grade 8 and decreased for grade 4. Concurrently, the proportion of items in Measurement in grade 8 decreased and the proportion in Number Properties and Operations in grade 4 increased.	Adjustments to the proportion of items on the assessment in Data Analysis, Statistics, and Probability at grades 4 and 8 reflect changes in opportunity to learn common across state standards. The distribution of attention to content topics in state standards informed the related decisions to increase the proportion of items at grade 4 in Number Properties and Operations and decrease the proportion in Measurement at grade 8. For more details on changes, see Chapter 2.
Mathematical Complexity (2017 Framework)	This was a chapter that defined mathematical complexity as "the demands on thinking that an item expects" (Governing Board, 2017a, p. 37). The chapter was removed.	From 2009 to 2017, "mathematical complexity" aimed to address the process dimension, the "doing" of knowing and doing mathematics, It was a mixing of cognitive demands (e.g., on working memory, reading comprehension, and attention) and the challenges inherent in developing mathematical understanding. However, it was not supportive of score interpretation. Many decades of research and development have shown that assessing students' knowledge and use of mathematics is more nuanced than was accounted for in the "mathematical complexity" approach used in previous frameworks.
NAEP Mathematical Practices (NEW)	A new chapter, Chapter 3 – NAEP Mathematical Practices, has been added describing and illustrating the assessment of five mathematical practices through which students engage in knowing and doing mathematics.	Since the 1990s, the field of mathematics education has seen increasing focus on mathematical processes and the interacting social and mental activities of knowing and doing mathematics. This chapter reflects the field's attention to mathematical activity by describing five NAEP Mathematical Practices. These are assessable aspects of activity at work across mathematics content when students do mathematics.

Exhibit 1.2. Summary of Changes (continued)

Topic	Change	Rationale
NAEP Mathematical Practices (NEW) (continued)	A distribution of items for each mathematical practice was developed.	Most NAEP Mathematics Assessment items will feature at least one of the five NAEP Mathematical Practices (55 to 85 percent). This range allows flexibility in assessment and item development across grades 4, 8, and 12 while also ensuring that the majority of the assessment is designed to capture information on student knowledge while engaging in mathematical practices. The balance of items (15 to 45 percent) will assess knowledge of content without calling on a particular mathematical practice (e.g., procedural or computational skill).
Item Formats and Assessment Design	Two chapters in the previous framework (Item Formats and Design of Test and Items) were merged into a single chapter, Chapter 4 – Overview of the Assessment Design, and updated.	The combination of chapters on assessment and item design allowed addressing interrelationships among: (1) the new digital format of NAEP administration, and (2) developments in technology for assessment, including scenario-based tasks.
	A new format, scenario-based task, was introduced.	With the addition of scenario-based tasks, the NAEP Mathematics Assessment continues to provide greater access to all students, diversifies the ways in which student achievement can be recognized and measured, and more robustly assesses both what students know and what they can do.

Exhibit 1.2. Summary of Changes (continued)

Topic	Change	Rationale
Calculator Policy	Continuing the policy established for the 2017 digital administration of NAEP, students will have access to a calculator emulator in blocks of items designated as "calculator blocks": four-function for grade 4, scientific for grade 8. The one change in 2026 and beyond will be that the grade 12 calculator will include a graphing emulator.	High school students typically use graphing calculators or online emulators and not scientific calculators (Crowe & Ma, 2010).
Item Types	Chapter 4 includes updates to reflect current and future digital platform use and the new format option of scenario-based tasks.	To better assess the diversity of ways of doing mathematics, technology available now and in the near future allows scenario-based tasks. Scenario-based item collections can be used to assess aspects of mathematical activity that have been difficult (if not impossible) to assess in the past. Building on the work in the last five years to use scenario-based tasks in NAEP Science and NAEP Technology and Engineering Literacy assessments, Chapter 4 details the ways scenario-based and traditional items can be combined to assess achievement in mathematics content and NAEP Mathematical Practices.
Tools and Manipulatives	Students will continue to have the tools and manipulatives used in the digital administration of the 2017 NAEP Mathematics Assessment. Chapter 4 also explores the potential of behind-the-scenes technology to capture and use process data for assessment; these are data generated by students as they work with the assessment.	The existing digital system tools and mathematics-specific tools have proven worthwhile since the 2017 administration. Additionally, in acknowledgment of the continuing evolution and use of technology in mathematics, Chapter 4 includes examples of other tools (e.g., simulations, dynamic geometry software, and "smart" physical objects) that may be common in 2026 and beyond.

MATHEMATICS CONTENT

The NAEP Mathematics Assessment measures what mathematics students know and are able to do, which involves understanding of particular mathematical ideas (content) and of how to use those ideas in mathematical activity (practices). The content of mathematics can be described by nouns: numbers, data, variables, functions, graphs, geometric figures of various kinds, and the like. In contrast, mathematical practices can be described by verbs: recognize, generalize, deduce, justify, and other processes of mathematical reasoning; represent, use, symbolize, and other actions involved in applying mathematics; describe, explain, model, and other activities inherent in mathematics being a discipline that is socially constructed by, and communicated among, individuals and societies.

This chapter focuses on the mathematics content objectives; Chapter 3 focuses on the NAEP Mathematical Practices. Mathematical proficiency involves knowing both.

Content Areas

NAEP has regularly gathered data on students' understanding of five broad areas of mathematics content:

- **Number Properties and Operations** (including computation and understanding of number concepts)
- **Measurement** (including use of instruments, application of processes, and concepts of area and volume)
- **Geometry** (including spatial reasoning and applying geometric properties)
- **Data Analysis, Statistics, and Probability** (including graphical displays)
- **Algebra** (including expressions, equations, representations, and relationships)

Classification of an item into one primary content area is not always clear-cut, but it helps to ensure that the indicated mathematical concepts and skills are assessed in a balanced way.

Certain aspects of mathematics occur in all content areas. For example, there is no single objective for computation. Instead, computation is embedded in many content objectives. In this framework, computation appears in the Number Properties and Operations objectives, which encompass a wide range of concepts about the numeration system and explicitly include a variety of computational skills, ranging from operations with whole numbers to work with decimals, fractions, percents, and real and complex numbers. Computation is also critical in Measurement and Geometry in determining, for example, the perimeter of a rectangle, estimating the height of a building, or finding the hypotenuse of a right triangle. Data analysis often involves computation in calculating a mean, or other statistics describing a collection of values, or in calculating probabilities. Solving algebraic equations also frequently involves numerical computation.

The objectives describe what is to be assessed on NAEP given operational limitations. As noted in Chapter 1, the NAEP content objectives are not a complete description of mathematics that should be taught at these grade levels.

NAEP Mathematics Assessment Objectives Terminology

Some terms that are broadly used in mathematics education must take on narrower meanings in order to clearly describe measurable mathematics objectives. To support item development aligned with the objectives given in this document, several points bear mention:

- The phrase "solve problems" means to complete tasks where the task contexts may range from the purely mathematical to those that are experientially concrete or real to students.
- When the word "or" is used in an objective, it means that an item may assess one or more of the concepts included, and the full collection of items will include assessment of each listed concept.
- Specific to grade 12 are three distinctions in NAEP content objectives:
    - Some grade 12 objectives are marked with an asterisk (*). This denotes objectives that describe mathematics content beyond what is typically taught in a 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra, with statistics and probability included). These objectives will be selected less often than the others for inclusion on the assessment.
    - Some objectives in grade 12 are marked with the number/hashtag sign (#). This designates objectives that most closely reflect opportunities to assess mathematical literacy. However, not all items associated with an objective that has the # sign will assess mathematical literacy.
    - At grade 12, geometry and measurement are combined as one content area. This reflects the fact that the majority of measurement topics suitable for high school students are geometric in nature.
- Although every assessment item will be assigned a primary classification, some items could potentially fall under more than one objective. Further clarification of objectives, along with sample items, can be found in the separate *Assessment and Item Specifications* document.

Mathematical Literacy

As noted in Chapter 1, mathematical literacy is related to an individual's capacity to "understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned citizen" (OECD, 2003, p. 3). It includes the ability to formulate and interpret problems, and to use mathematical knowledge and skill in creative ways across a range of situations—complex and simple, routine and unusual. These situations can occur in one's private life (measuring cloth for a project), one's occupational and professional life (using proportions to make sense of a situation), one's social life with friends or family (paying in a restaurant), and in one's life as a citizen (processing information relevant to voting).

Some objectives at grade 12 are identified with the theme of mathematical literacy. If there are everyday applications of the objective to situations in a person's life as a community member, citizen, worker, or consumer, then the number/hashtag sign (#) precedes the objective. For example, for an objective that calls for students to analyze situations, develop mathematical models, or solve problems using a particular form of equation or inequality, mathematical literacy items might be given in real-world contexts such as solving a problem about tax implications of a workplace policy change, or, in the context of community decisions, analyzing or modeling with an inequality the upper bounds for safe levels of lead in water from a local water treatment facility. Other items *not* focused on mathematical literacy might ask the student to solve a problem by graphing the consequences of doubling the value of a variable in a linear relationship.

As another example, a mathematical literacy assessment item might provide information about a seismic magnitude scale (used to measure the intensity of earthquakes), indicate that on the scale a Magnitude 5 earthquake is ten times stronger than a Magnitude 4 earthquake, and ask grade 12 students to make sense of, model, or draw conclusions in a problem situation that uses that information. An alternate assessment item for the same objective that would not be focused on mathematical literacy might ask students to apply and justify the use of logarithms to determine the seismic magnitude measurement in a given situation. The goal of the identification of objectives with # is to support exploration of NAEP reporting on mathematical literacy. See the *Assessment and Item Specifications* document for a description of a special study on assessing and reporting on mathematical literacy.

Item Distribution

The distribution of items among the various mathematics content areas is a critical feature of the assessment design because it reflects the relative importance given to each area in the assessment. As has been the case with past NAEP assessments, the categories have different emphases at each grade. Exhibit 2.1 provides the balance of items in the assessment by content area for each grade (4, 8, and 12). The percentages refer to the proportion of items, not the amount of testing time.

For the 2026 NAEP Mathematics Assessment, a greater number of items assessing fraction concepts will be sampled than have been in past administrations. This increase reflects not only the focus on fraction instruction in the early grades, but also the importance of understanding students' early knowledge of and skills with fraction concepts, as they are a predictor of success in high school mathematics courses (Siegler et al., 2012).

Exhibit 2.1. Percentage Distribution of Items by Grade and Content Area

<table>
<tr><th>Content Area</th><th>Grade 4</th><th>Grade 8</th><th>Grade 12</th></tr>
<tr><td>Number Properties and Operations</td><td>45*</td><td>20</td><td>10</td></tr>
<tr><td>Measurement</td><td>20</td><td>10</td><td rowspan="2">30</td></tr>
<tr><td>Geometry</td><td>15</td><td>20</td></tr>
<tr><td>Data Analysis, Statistics, and Probability</td><td>5</td><td>20</td><td>25</td></tr>
<tr><td>Algebra</td><td>15</td><td>30</td><td>35</td></tr>
</table>

*Note: At least one-third of grade 4 Number Properties and Operations items should assess fraction content.

NAEP Mathematics Objectives Organization

Mathematical ideas in different content areas are often interconnected. Organizing this framework by content areas has the potential for obscuring these connections and leading to fragmentation. However, the intent here is that the objectives and the assessment of those objectives will, in many cases, cross content area boundaries.

To provide clarity and specificity in grade-level objectives, the framework matrix (Exhibits 2.2, 2.3, 2.4, 2.5, and 2.6) depicts the objectives appropriate for assessment under each subtopic. For example, within the Number Properties and Operations subtopic of Number Sense, specific objectives are listed for assessment at grades 4, 8, and 12. In general, objectives within content areas are different across the grades. Occasionally, the same objective may appear at more than one grade level; this suggests an implicit developmental sequence for that concept or skill. An empty cell in the matrix conveys that an objective is not appropriate or not deemed as important as other areas for assessment at that grade level. Explanations of changes in the mathematics objectives are elaborated in the final section of this chapter.

Number Properties and Operations

Numbers (used as counts, measures, ratio comparisons, and scale values) are tools for describing the world quantitatively. It is thus not surprising that *Number* constitutes a major content focus of school mathematics, especially through grade 8. This focus includes facility with different notational forms (as whole numbers, fractions, decimals, percents, powers, and radicals), an understanding of number systems (e.g., integers, rational numbers, real numbers) and their properties, and calculational proficiency with these forms within systems.

Ancient cultures around the world had names for numbers and ways of doing arithmetic. The accessibility and usefulness of arithmetic today is greatly enhanced by the worldwide use of the Hindu-Arabic decimal place value system. In its full development, this remarkable system includes finite and infinite decimals that allow approximating any real number as closely as desired. Decimal notation simplifies arithmetic by means of routine algorithms; it makes size comparisons straightforward and estimation simple.

Numbers are not simply labels for quantities; they form systems with their own internal structure. For instance, at times problems can be more easily solved by considering what numbers add up to a certain value (e.g., 100 – 98 can be thought of as "98 plus what adds up to 100?"). Multiplication is connected to the idea of repeated addition just as division is connected to the idea of repeated subtraction, and the relationship between multiplication and division can be used to simplify computation (e.g., instead of multiplying a number by 25, a number can be multiplied by 100 and then divided by 4, perhaps by halving and halving again). Arithmetic operations (addition, subtraction, multiplication, and division) and the relationships among them help students determine the mathematics that corresponds to basic real-world actions. For example, joining two collections or laying two lengths end-to-end can be described by addition, while comparing two collections can be described by subtraction, and the concept of rate depends on division. Multiplication and division of whole numbers lead to the beginnings of number theory, including concepts of factorization, remainder, and prime number. Another basic structure of real numbers is ordering, as in which is greater or lesser. Attention to the relative size of quantities provides a basis for making sensible estimates.

Number is not an isolated mathematics domain; it is intimately interwoven with other content strands. In their study of measurement, students use numbers to describe continuous quantities such as length, area, volume, weight, and time, and even to describe more complicated derived quantities such as rates of speed, density, inflation, interest, and so on. With numbers, students can count collections of discrete objects or describe fractional parts of data sets, allowing for statistical analysis. As elementary-grade students generalize number relationships and properties they engage in algebraic thinking. In pursuit of graphical depictions of algebraic relationships, students use Cartesian coordinates—ordered pairs of numbers to identify points in a plane and ordered triples of numbers to label points in space. Numbers allow precise communication about anything that can be counted, measured, or located in space.

Comfort in dealing with numbers effectively is called *number sense*. It includes intuition about what numbers mean; understanding the ways to represent numbers symbolically (including facility with converting between different representations); the ability to calculate, either exactly or approximately, and by several methods (e.g., mentally, with paper and pencil, or calculator, as appropriate); and the ability to estimate. Skill in working with proportions (including percents) is another important part of number sense.

Number sense is a major expectation of the NAEP Mathematics Assessment. In grade 4, students are expected to have a solid grasp of whole numbers as represented in the base 10 system and to begin understanding fractions. By grade 8, students should be comfortable with rational numbers, represented either as decimal fractions or as common fractions, and should be able to use them to solve problems involving proportionality, percentages, and rates. At this level, number sense should also begin to coalesce with geometry by extending students' understanding of the number line. This concept is connected with approximation and the use of scientific notation. Grade 8 students should also have some acquaintance with naturally occurring irrational numbers, such as square roots and π (pi). By grade 12, students should be comfortable dealing with all types of real numbers and various representations, for example, as powers. Students in grade 12 should be able to establish the validity of numerical properties using mathematical arguments. The 2026 Number Properties and Operations objectives are shown in Exhibit 2.2.

Exhibit 2.2. Number Properties and Operations (Num)

Num – 1. Number sense		
Grade 4	**Grade 8**	**Grade 12**
a) Identify place value and actual value of digits in whole numbers, and think flexibly about place value notions (e.g., there are 2 hundreds in 253, there are 25 tens in 253, there are 253 ones in 253).	a) Use place value to represent and describe integers and decimals.	
b) Represent numbers using base 10, number line, and other representations.	b) Represent or describe rational numbers or numerical relationships using number lines and diagrams.	
c) Compose or decompose whole quantities either by place value (e.g., write whole numbers in expanded notation using place value: $342 = 300 + 40 + 2$ or $3 \times 100 + 4 \times 10 + 2 \times 1$) or convenience (e.g., to compute 4×27 decompose 27 into $25 + 2$ because 4×25 is 100, and 4×2 is 8 so 4×27 is 108).		
d) Write or rename whole numbers (e.g., 10: $5 + 5$, $12 – 2$, 2×5).	d) Write or rename rational numbers.	# d) Represent, interpret, or compare expressions for real numbers, including expressions using exponents and *logarithms.
e) Connect across various representations for whole numbers, fractions, and decimals (e.g., number word, number symbol, visual representations).	e) Recognize, translate, or apply multiple representations of rational numbers (fractions, decimals, and percents) in meaningful contexts.	
	f) Express or interpret large numbers using scientific notation from real-life contexts.	# f) Represent or interpret expressions involving very large or very small numbers in scientific notation.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

\# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.2. Number Properties and Operations (continued)

Num – 1. Number sense (continued)		
Grade 4	**Grade 8**	**Grade 12**
	g) Find absolute values or apply them to problem situations.	g) Represent, interpret, or compare expressions or problem situations involving absolute values.
h) Recognize and generate simple equivalent (equal) fractions and explain why they are equivalent (e.g., by using drawings).	h) Order or compare rational numbers (fractions, decimals, percents, or integers) using various representations (e.g., number line).	
i) Order or compare whole numbers, decimals, or fractions using common denominators or benchmarks.	i) Order or compare rational numbers including very large and small integers, and decimals and fractions close to zero.	# i) Order or compare rational or irrational numbers, including very large and very small real numbers.

Num – 2. Estimation		
Grade 4	**Grade 8**	**Grade 12**
a) Use benchmarks (well-known numbers used as meaningful points for comparison) for whole numbers, decimals, or fractions in contexts (e.g., ½ and 0.5 may be used as benchmarks for fractions and decimals between 0 and 1.00).	a) Establish or apply benchmarks for rational numbers and common irrational numbers (e.g., π) in contexts.	
b) Make estimates appropriate to a given situation with whole numbers, fractions, or decimals.	b) Make estimates appropriate to a given situation by: • Identifying when estimation is appropriate, • Determining the level of accuracy needed, • Selecting the appropriate method of estimation.	# b) Identify situations where estimation is appropriate, determine the needed degree of accuracy, and *analyze the effect of the estimation method on the accuracy of results.
c) Verify and defend solutions or determine the reasonableness of results in meaningful contexts.	c) Verify solutions or determine the reasonableness of results in a variety of situations, including calculator or computer results.	# c) Verify solutions or determine the reasonableness of results in a variety of situations.
	d) Estimate square or cube roots of numbers less than 150 between two whole numbers.	d) Estimate square or cube roots of numbers less than 1,000 between two whole numbers.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).
# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.2. Number Properties and Operations (continued)

Num – 3. Number operations		
Grade 4	**Grade 8**	**Grade 12**
a) Add and subtract using conventional or unconventional procedures (e.g., strategic decomposing and composing): • Whole numbers, or • Fractions and mixed numbers with like denominators.	a) Perform computations with rational numbers.	a) Find integer or simple rational powers of real numbers.
b) Multiply numbers using conventional or unconventional procedures (e.g., strategic decomposing and composing): • Whole numbers no larger than two digits by two digits with paper and pencil computation, or • Larger whole numbers using a calculator, or • Multiplying a fraction by a whole number.		b) Perform arithmetic operations with real numbers, including common irrational numbers.
c) Divide whole numbers: • Up to three digits by one digit with paper and pencil computation, or • Up to five digits by two digits with use of calculator.		c) Perform arithmetic operations with expressions involving absolute value.
	d) Describe the effect of operations on size, including the effect of attempts to multiply or divide a rational number by: • Zero, or • A number less than zero, or • A number between zero and one, or • One, or • A number greater than one.	d) Describe the effect of multiplying and dividing by numbers including the effect of attempts to multiply or divide a real number by: • Zero, or • A number less than zero, or • A number between zero and one, or • One, or • A number greater than one.

Exhibit 2.2. Number Properties and Operations (continued)

Num – 3. Number operations (continued)		
Grade 4	**Grade 8**	**Grade 12**
e) Interpret, explain, or justify whole number operations and explain the relationships between them.	e) Interpret, explain, or justify rational number operations and explain the relationships between them.	e) *Analyze or interpret a proof by mathematical induction of a simple numerical relationship.
f) Solve problems involving whole numbers and fractions with like denominators.	f) Solve problems involving rational numbers and operations using exact answers or estimates as appropriate.	# f) Solve problems involving numbers, including rationals and common irrationals.

Num – 4. Ratios and proportional reasoning		
Grade 4	**Grade 8**	**Grade 12**
	a) Use ratios to describe problem situations.	
	b) Use fractions to represent and express ratios and proportions.	
	c) Use proportional reasoning to model and solve problems (including rates and scaling).	# c) Use proportions to solve problems (including rates of change and per capita problems).
	d) Solve problems involving percentages (including percent increase and decrease, interest rates, tax, discount, tips, or part/whole relationships).	# d) Solve multistep problems involving percentages, including compound percentages.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).
# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.2. Number Properties and Operations (continued)

Num – 5. Properties of number and operations		
Grade 4	**Grade 8**	**Grade 12**
a) Identify odd and even numbers.		
b) Identify factors of whole numbers.	b) Recognize, find, or use factors, multiples, or prime factorization.	
	c) Recognize or use prime and composite numbers to solve problems.	c) Solve problems using factors, multiples, or prime factorization.
	d) Use divisibility or remainders in problem settings.	# d) Use divisibility or remainders in problem settings.
e) Apply basic properties of operations.	e) Apply basic properties of operations, including conventions about the order of operations as applied to integers and rational numbers.	e) Apply basic properties of operations, including conventions about the order of operations as applied to real numbers.
		f) Recognize properties of the number system (whole numbers, integers, rational numbers, real numbers, and *complex numbers) and how they are related to each other and identify examples of each type of number.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Measurement

Measuring is the process by which numbers are assigned to describe the world quantitatively. This process involves selecting the attribute of the object or event to be measured, comparing this attribute to a unit, and reporting the number of units. For example, in measuring a banner, one may select the attribute of length and the inch as a unit for the comparison. In comparing lengths to the nearest inch, it may be that a length is *about* 42 inches. If considering only the domain of whole numbers, one would report that the banner is 42 inches long. However, because length is a continuous attribute, in the domain of rational numbers the length of the banner might be reported as $41^{13}/_{16}$ inches (to the nearest 16^{th} of an inch) or 41.8 inches (to the nearest 0.1 inch).

The connection between measuring and number makes measurement a vital part of school mathematics. Measurement is an important setting for negative and irrational numbers as well as positive numbers, since negative numbers arise naturally from situations with two directions and irrational numbers are commonplace in geometry. Measurement representations and tools are often used when students are learning about number properties and operations. For example, area grids and representations of volume using unit cubes can help students understand multiplication and its properties. The number line can help students understand ordering and rounding numbers. Measurement also has a strong connection to other areas of school mathematics and other subjects. Problems in algebra are often drawn from measurement situations and functions are used to relate measures to each other. Geometry regularly focuses on measurement aspects of geometric figures. Probability and statistics provide ways to measure chance and to compare sets of data. The measurement of time, values of goods and services, physical properties of objects, distances, and various kinds of rates exemplify the importance of measurement in everyday activities.

In this framework, attributes such as capacity, weight, mass, time, and temperature are included, as are the geometric attributes of length, area, and volume. Many of these attributes appear in grade 4, where the emphasis is on length, including perimeter, distance, and height. More emphasis is placed on area and angle measure in grade 8. By grade 12, measurement in everyday life, as well as in the study of volumes and rates constructed from other attributes, such as speed, is emphasized.

The 2026 NAEP Mathematics Assessment includes nonstandard, customary, and metric units. At grade 4, common customary units such as inch, quart, pound, hour, and degree (for measuring angles) are included, and common metric units such as centimeter, liter, and gram are emphasized. Grades 8 and 12 include the use of both square and cubic units for measuring area, surface area, and volume; continued use of degrees for measuring angles; and constructed units such as miles per hour. Converting from one unit in a system to another, such as from minutes to hours, is an important aspect of measurement included in problem situations. Understanding and using the many conversions available is an important skill. There are a limited number of common, everyday equivalencies that students are expected to know (see the *Assessment and Item Specifications* document for more detail).

Items classified in this content area depend on some knowledge of measurement. For example, an item comparing a 2-foot segment with an 8-inch line segment is classified as a measurement item, whereas an item that asks for the difference between a 3-inch and a 1¾-inch line segment would be classified as a number item. In many secondary schools, measurement becomes an integral part of geometry, and this is reflected in the proportion of items recommended for these two areas (see Exhibit 2.1). The 2026 Measurement objectives are shown in Exhibit 2.3.

Exhibit 2.3. Measurement (Meas)

Meas – 1. Measuring physical attributes		
Grade 4	**Grade 8**	**Grade 12**
a) Identify the attribute that is appropriate to measure in a given situation.		
b) Compare objects with respect to a given attribute, such as length, area, capacity, time, or temperature.	b) Compare objects with respect to length, area, volume, angle measurement, weight, or mass.	# b) Determine the effect of proportions and scaling on length, area, and volume.
c) Estimate the size of an object with respect to a given measurement attribute (e.g., length, perimeter, or area using a grid).	c) Estimate the size of an object with respect to a given measurement attribute (e.g., area).	# c) Estimate or compare perimeters or areas of two-dimensional geometric figures.
		d) Solve problems of angle measure, including those involving triangles or other polygons or parallel lines cut by a transversal.
e) Select or use appropriate measurement instruments such as ruler, meter stick, clock, thermometer, or other scaled instruments.	e) Select or use appropriate measurement instruments to determine or create a given length, area, volume, angle, weight, or mass.	
f) Solve problems involving perimeter of plane figures.	f) Solve mathematical or real-world problems involving perimeter or area of plane figures such as triangles, rectangles, circles, or composite figures.	f) Solve problems involving perimeter or area of plane figures such as polygons, circles, or composite figures.
g) Solve problems involving area of squares and rectangles.		
	h) Solve problems involving volume or surface area of rectangular solids, and volume of right cylinders and prisms, or composite shapes.	# h) Solve problems by determining, estimating, or comparing volumes or surface areas of three-dimensional figures.
	i) Solve problems involving rates and ratios such as speed or population density.	# i) Solve problems involving rates and ratios such as speed, density, population density, or flow rates.

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.3. Measurement (continued)

Meas – 2. Systems of measurement		
Grade 4	**Grade 8**	**Grade 12**
a) Select or use an appropriate type of unit for the attribute being measured such as length, angle size, time, or temperature.	a) Select or use an appropriate type of unit for the attribute being measured such as length, area, angle, time, or volume.	# a) Choose appropriate units for geometric measurements (length, area, perimeter, volume) and apply units in expressions, equations, and problem solutions.
b) Solve problems involving conversions within the same measurement system such as conversions involving inches and feet or hours and minutes.	b) Solve problems involving conversions within the same measurement system such as conversions involving square inches and square feet.	# b) Solve problems involving conversions within or between measurement systems, given a relationship between the units.
	c) Estimate the measure of an object in one system given the measure of that object in another system and the approximate conversion factor. For example: • Distance: 1 kilometer is approximately 0.6 mile. • Money: U.S. dollars to Canadian dollars. • Temperature: Fahrenheit to Celsius.	
d) Determine appropriate unit of measurement in problem situations involving such attributes as length, time, capacity, or weight.	d) Determine appropriate unit of measurement in problem situations involving such attributes as length, area, or volume.	# d) Understand that numerical values associated with measurements of physical quantities are approximate, subject to variation, and must be assigned units of measurement.
		# e) Determine appropriate accuracy of measurement in problem situations (e.g., the accuracy of measurement of the dimensions to obtain a specified accuracy of area) and find the measure to that degree of accuracy.
	f) Construct or solve problems (e.g., floor area of a room) involving scale drawings.	# f) Construct or solve problems involving scale drawings.

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.3. Measurement (continued)

Meas – 3. Measurement in triangles		
Grade 4	**Grade 8**	**Grade 12**
		# a) Solve problems involving indirect measurement.
		b) Solve problems using the fact that trigonometric ratios (sine, cosine, and tangent) stay constant in similar triangles.
		c) Use the definitions of sine, cosine, and tangent as ratios of sides in a right triangle to solve problems about length of sides and measure of angles.
		d) * Interpret and use the identity $\sin^2\theta + \cos^2\theta = 1$ for angles θ between 0° and 90°; recognize this identity as a special representation of the Pythagorean theorem.
		e) * Determine the radian measure of an angle and explain how radian measurement is related to a circle of radius 1.
		f) * Use trigonometric formulas such as addition and double angle formulas.
		g) * Use the law of cosines and the law of sines to find unknown sides and angles of a triangle.
		h) * Interpret the graphs of the sine, cosine, and tangent functions with respect to periodicity and values of these functions for multiples of $\pi/6$ and $\pi/4$.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Geometry

Geometry began thousands of years ago in many lands as sets of practical rules related to describing and predicting locations of astronomical objects, calculating land areas, and building structures. More than 2,200 years ago, the Greek mathematician Euclid organized the geometry known at that time into a coherent collection of results, all deduced using logic from a small number of postulates assumed to be true. Euclid's work was fundamental in establishing mathematical truth as dependent on valid deductive reasoning rather than reliant on educated guesses from several specific examples. The theorems obtained via deduction by Euclid remain fundamental to the study of geometry, and for this reason the geometry studied in school is called Euclidean geometry.

The fundamental concepts of Euclidean geometry are congruence, similarity, and symmetry. By grade 4, students are expected to be familiar with a library of simple figures and their attributes, both in the plane (lines, circles, triangles, squares, and rectangles) and in space (cubes, spheres, and cylinders).

By grade 8, understanding of these shapes deepens, with study of cross sections of solids and the beginnings of an analytical understanding of properties of plane figures, especially parallelism, perpendicularity, and angle relations in polygons. Reflections, translations, and rotations (mathematical models of the physical phenomena of reflecting, sliding, and turning) are introduced as distance-preserving transformations that map a figure onto a congruent image. Dilatations (expansions and contractions) map figures onto similar images. Properties of congruent and similar figures involve angle measures and lengths, so geometry becomes more and more mixed with measurement in later grades. Placing figures on a coordinate plane provides the beginnings of the connections among algebra, geometry, and analytic geometry.

In secondary school, the content of plane geometry is logically ordered, and students are expected to make, test, and validate conjectures. Students see that most of the commonly studied plane figures—triangles (scalene, isosceles, equilateral) and quadrilaterals (parallelogram, rectangle, rhombus, square, trapezoid)—may possess reflection or rotation symmetry, or both, and can use triangle congruence and similarity theorems as well as symmetry to establish properties of figures. By grade 12, students may also gain insight into systematic structure, such as the classification of distance-preserving transformations of the plane (that is, reflections, rotations, translations, or glide reflections), and what happens when two or more isometries are performed in succession (composition). In analytic geometry, the key areas of geometry and algebra merge into a powerful tool that provides a basis for calculus and much of applied mathematics. The 2026 Geometry objectives are shown in Exhibit 2.4.

Exhibit 2.4. Geometry (Geom)

Geom – 1. Dimension and shape		
Grade 4	**Grade 8**	**Grade 12**
a) Identify or describe (informally) real-world objects using simple plane figures (e.g., triangles, rectangles, squares, and circles) and simple solid figures (e.g., cubes, spheres, and cylinders).	a) Identify a geometric object given a written description of its properties.	
b) Identify or draw angles and other geometric figures in the plane.	b) Identify, define, or describe geometric shapes in the plane and in three-dimensional space given a visual representation.	b) Give precise mathematical descriptions or definitions of geometric shapes in the plane and in three-dimensional space.
	c) Draw or sketch from a written description polygons, circles, or semicircles.	c) Draw or sketch from a written description plane figures and planar images of three-dimensional figures.
		# d) Use two-dimensional representations of three-dimensional objects to visualize and solve problems.
e) Describe or distinguish among attributes of two- and three-dimensional shapes.	e) Demonstrate an understanding of two- and three-dimensional shapes in the world through identifying, drawing, reasoning from visual representations, composing, or decomposing.	# e) Analyze properties of three-dimensional figures including prisms, pyramids, cylinders, cones, spheres, and hemispheres.

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.4. Geometry (continued)

Geom – 2. Transformation of figures and preservation of properties		
Grade 4	**Grade 8**	**Grade 12**
	a) Identify lines of symmetry in plane figures or recognize and classify types of symmetries of plane figures.	a) Recognize or identify types of symmetries (e.g., translation, reflection, rotation) of two- and three-dimensional figures.
		b) Give or recognize the precise mathematical relationship (e.g., congruence, similarity, orientation) between a figure and its image under a transformation.
	c) Recognize or informally describe the effect of a transformation (reflection, rotation, translation, or dilation) on two-dimensional figures.	c) Perform or describe the effect of a single transformation (reflection, rotation, translation, or dilation) on two- or three-dimensional geometric figures.
d) Recognize attributes (such as shape and area) that do not change when plane figures are subdivided and rearranged.	d) Predict results of combining, subdividing, and recombining shapes of plane figures and solids (e.g., paper folding, tiling, subdividing and rearranging the pieces).	d) Identify transformations of shapes that preserve the area of two-dimensional figures or the volume of three-dimensional figures.
	e) Justify relationships of congruence and similarity and apply these relationships using scaling and proportional reasoning.	e) Justify relationships of congruence and similarity and apply these relationships using scaling, proportional reasoning, and established theorems.
	f) Apply the relationships among angle measures, lengths, and perimeters among similar figures.	f) Apply the relationships among angle measures, lengths, perimeters, and volumes among similar figures.
		g) Perform or describe the effects of successive (composites of) isometries and/or similarity transformations.

Exhibit 2.4. Geometry (continued)

Geom – 3. Relationships between geometric figures		
Grade 4	**Grade 8**	**Grade 12**
a) Analyze or describe patterns in polygons when the number of sides increases, or the size or orientation changes.		
b) Combine simple plane shapes to construct a given shape.	b) Apply geometric properties and relationships in solving problems in two and three dimensions.	b) Apply geometric properties and relationships to solve problems in two and three dimensions.
c) Recognize two-dimensional faces of three-dimensional shapes.	c) Represent problem situations with geometric figures to solve problems.	# c) Represent problem situations with geometric figures to solve mathematical or real-world problems.
	d) Use the Pythagorean theorem to solve problems in two-dimensional situations.	# d) Use the Pythagorean theorem to solve problems in two- or three-dimensional situations.
		e) Recall and interpret or use definitions and basic properties of congruent and similar triangles, quadrilaterals, and other polygons; circles; parallel, perpendicular, and intersecting lines; and associated angle relationships (e.g., in solving problems or creating proofs).
f) Describe and compare attributes of simple and compound figures composed of triangles, squares, and rectangles.	f) Describe, compare, or analyze attributes of, or relationships between, triangles, quadrilaterals, and other polygonal plane figures.	f) Analyze attributes or relationships of triangles, quadrilaterals, and other polygonal plane figures.
	g) Describe or analyze properties and relationships of parallel or intersecting lines.	g) Analyze properties and relationships of parallel, perpendicular, or intersecting lines, including the angle relationships that arise in these cases.

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.4. Geometry (continued)

Geom – 3. Relationships between geometric figures (continued)		
Grade 4	**Grade 8**	**Grade 12**
		h) Make, test, and validate geometric conjectures using a variety of methods, including deductive reasoning and counterexamples.
		i) * Analyze properties of circles and the intersections of lines and circles (inscribed angles, central angles, tangents, secants, and chords).

Geom – 4. Position, direction, and coordinate geometry		
Grade 4	**Grade 8**	**Grade 12**
a) Describe relative positions of points and lines using the geometric ideas of parallelism or perpendicularity.	a) Describe relative positions of points and lines using the geometric ideas of midpoint, points on a common line through a common point, parallelism, or perpendicularity.	a) Solve problems involving the coordinate plane using distance between two points, the midpoint of a segment, or slopes of perpendicular or parallel lines.
	b) Describe the intersection of two or more geometric figures in the plane (e.g., intersection of a circle and a line).	b) Describe the intersections of lines in the plane and in space, of a line and a plane, or of two planes in space.
	c) Visualize or describe the cross section of a solid.	c) Describe or identify conic sections and other cross sections of solids.
	d) Represent geometric figures using rectangular coordinates on a plane.	d) Represent two-dimensional figures algebraically using coordinates and/or equations.
		e) * Use vectors to represent velocity and direction; multiply a vector by a scalar and add vectors both algebraically and graphically.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Exhibit 2.4. Geometry (continued)

Geom – 4. Position, direction, and coordinate geometry (continued)		
Grade 4	**Grade 8**	**Grade 12**
		f) Find an equation of a circle given its center and radius and, given an equation of a circle, find its center and radius.
		g) * Graph or determine equations for images of lines, circles, parabolas, and other curves under translations and reflections in the coordinate plane.
		h) * Represent situations and solve problems involving polar coordinates.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Data Analysis, Statistics, and Probability

Data analysis and statistics refers to the entire process of collecting, organizing, summarizing, and interpreting data. This is the heart of statistics and is in evidence whenever quantitative information is used to determine a course of action. Data analysis normally begins with a question to be answered. Statistical questions can arise prior to data collection, or from existing data sets. Beginning at an early age, students should grasp the fundamental principle that exploratory data analysis of an existing data set is far different from the scientific method of collecting data to verify or refute a well-posed question. Data can be useful when collected with a specific question in mind and when there is a plan (usually called a *design*) for using the data to answer the question. However, contemporary uses of data-mining techniques associated with "big data" suggest that data sets may subsequently be useful in answering questions that were not envisioned when the data collection was initiated.

A probability is a measure of uncertainty. This measure may be determined from a theoretical model that makes assumptions about equally likely or weighted outcomes for an event (as when one says that the probability of a coin landing head-side up is one-half) or it may be determined in some way from past experience, as when forecasters say the probability of rain tomorrow is 40 percent. Statistical analysis often involves studying whether assumptions about theoretical probability match observed relative frequencies. For instance, if a coin tossed 100 times turned up heads 80 times, one might suspect that the probability of heads for that coin is not ½ (the theoretical probability of heads for a fair coin). Under random sampling, patterns for outcomes of designed studies can be anticipated and used as a basis for making decisions. The probability distribution of all possible outcomes is important in most statistical decision-making because the key is to decide whether or not a particular observed outcome is typical or unusual (located in a tail of a probability distribution). For example, 4.0 as a grade-point average is unusually high among most student groups, 4 as the weight in pounds of a human baby is unusually low, and 4 as the number of floors in a building is not unusual in either direction.

By grade 4, students are expected to apply their understanding of number and quantity to consider questions that can be answered by examining appropriate data. Building on the principles of describing data distributions through minimum, maximum, and clusters of values, grade 8 students are expected to use a wider variety of organizing and summarizing techniques for center, spread, and shape. They can identify and construct a statistical question, one that needs data in order to be addressed. They can also begin to analyze statistical claims through designed surveys and experiments that involve randomization. Also by grade 8, students are expected to begin to use more formal terminology related to probability and data analysis. They can identify associations between two numerical variables in scatterplots, as well as the relative strength of those associations.

Grade 12 students are expected to use a wide variety of statistical techniques for all phases of data analysis, including a more formal understanding of statistical inference, and simulation as an inferential analysis tool. In addition to comparing univariate data sets, students at this level can recognize and describe possible associations between two variables by looking at two-way tables for categorical variables or scatterplots for measurement variables. By grade 12, students should be able to use linear equations to describe possible associations between measurement variables and should be familiar with techniques for fitting functions to data. The 2026 Data Analysis, Statistics, and Probability objectives are shown in Exhibit 2.5.

Exhibit 2.5. Data Analysis, Statistics, and Probability (Data)

Data – 1. Data representation		
Grade 4	**Grade 8**	**Grade 12**
Representations of data are indicated for each grade level in the next row. For some objectives, only a subset of the representations is applicable, indicated by a parenthetical list at the end of the objective.		
Pictographs, bar graphs, dot plots, tables, and tallies.	Histograms, plots over time, dot plots, scatterplots, box plots, bar graphs, circle graphs, stem and leaf plots, frequency distributions, and tables.	Histograms, plots over time, dot plots, scatterplots, box plots, bar graphs, circle graphs, stem and leaf plots, frequency distributions, and tables, including two-way tables.
a) Read or interpret a single distribution of data.	a) Read or interpret data, including interpolating or extrapolating from data.	# a) Read or interpret graphical or tabular representations of data.
b) For a given distribution of data, complete a graph (limits of time make it difficult to construct graphs completely).	b) For a given distribution of data, complete a graph and solve a problem using the data in the graph (histograms, plots over time, dot plots, scatterplots, bar graphs, circle graphs).	# b) For a given set of data, complete a graph and solve a problem using the data in the graph (histograms, plots over time, dot plots, scatterplots).
c) Answer statistical questions by estimating and computing within a single distribution of data.	c) Answer statistical questions by estimating and computing with data from a single distribution or across distributions of data.	c) Answer statistical questions involving univariate or bivariate distributions of data.
	d) Given a graphical or tabular representation of a distribution of data, determine whether the information is represented effectively and appropriately (histograms, plots over time, dot plots, scatterplots, box plots, bar graphs, circle graphs).	# d) Analyze, compare, and contrast different graphical representations of univariate and bivariate data (e.g., identify misleading uses of data in real-world settings and critique different ways of presenting and using information).
		# e) * Organize and display data in a spreadsheet in order to recognize patterns and solve problems.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

\# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.5. Data Analysis, Statistics, and Probability (continued)

Data – 2. Characteristics of data sets		
Grade 4	**Grade 8**	**Grade 12**
	a) Calculate, use, or interpret mean, median, mode, range, or shape of a distribution of data.	# a) Calculate, interpret, or use summary statistics for distributions of data including measures of center (mean, median), position (quartiles, percentiles), spread (range, interquartile range, variance, and standard deviation) or shape (skew, uniform, uni-/bimodal).
b) Given a distribution of whole number data in a context, identify and explain the meaning of the greatest value, of the least value, or of any clustering or grouping of data in the distribution.	b) Describe a distribution of data using its mean, median, mode, range, interquartile range, and shape.	b) Recognize how linear transformations of one-variable data affect mean, median, mode, range, interquartile range, and standard deviation.
	c) Identify outliers and determine their effect on the mean, median, mode, or range.	# c) Determine the effect of outliers on the mean, median, mode, range, interquartile range, or standard deviation.
	d) Using appropriate statistical measures, compare two or more data sets describing the same characteristic for two different populations or subsets of the same population.	# d) Compare data sets using summary statistics (mean, median, mode, range, interquartile range, shape, or standard deviation) describing the same characteristic for two different populations or subsets of the same population.
	e) Visually choose the line that best fits given a scatterplot and informally explain the meaning of the line. Use the line to make predictions.	e) Approximate a trend line if a linear pattern is apparent in a scatterplot or use a graphing calculator to determine a least-squares regression line and use the line or equation to make predictions.

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.5. Data Analysis, Statistics, and Probability (continued)

Data – 2. Characteristics of data sets (continued)		
Grade 4	**Grade 8**	**Grade 12**
		# f) Recognize or explain how an argument based on data might confuse correlation with causation.
		g) * Identify and interpret the key characteristics of a normal distribution such as shape, center (mean), and spread (standard deviation).
		# h) * Recognize and explain the potential errors that can arise when extrapolating from data.

Data – 3. Experiments and samples		
Grade 4	**Grade 8**	**Grade 12**
	a) Given a sample, identify possible sources of bias in sampling.	# a) Identify possible sources of bias in sample survey populations or questions and describe how such bias can be controlled and reduced.
	b) Distinguish between a random sample and a nonrandom sample.	b) Recognize and describe a method to select a simple random sample.
		# c) Draw inferences from samples, such as estimates of proportions in a population, estimates of population means, or decisions about differences in means for two “treatments.”
		d) Identify or evaluate the characteristics of a good survey or of a well-designed experiment.
		e) * Recognize the differences in design and in conclusions between randomized experiments and observational studies.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

\# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.5. Data Analysis, Statistics, and Probability (continued)

Data – 4. Probability		
Grade 4	**Grade 8**	**Grade 12**
		# a) Determine whether two events are independent or dependent.
	b) Using assumption of randomness, determine the theoretical probability of simple or compound events in familiar contexts.	# b) Using assumptions such as randomness, determine the theoretical probability of simple or compound events in familiar or unfamiliar contexts.
	c) Given the results of an experiment or simulation, estimate the probability of simple and compound events in familiar contexts.	# c) Given the results of an experiment or simulation, estimate the probability of simple or compound events in familiar or unfamiliar contexts.
	d) Use theoretical probability to evaluate or predict experimental outcomes in familiar contexts.	# d) Use theoretical probability to evaluate or predict experimental outcomes in familiar or unfamiliar contexts.
	e) Determine the sample space for a given situation.	e) Determine the number of ways an event can occur using tree diagrams, formulas for combinations and permutations, or other counting techniques.
	f) Use a sample space to determine the probability of possible outcomes for an event.	
	g) Represent the probability of a given outcome using fractions, decimals, and percents.	
	h) Determine the probability of independent and dependent events (dependent events should be limited to a small sample size).	h) Determine the probability of independent and dependent events.

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.5. Data Analysis, Statistics, and Probability (continued)

Data – 4. Probability (continued)		
Grade 4	**Grade 8**	**Grade 12**
		i) Determine conditional probability using two-way tables.
	j) Interpret and apply probability concepts to practical situations, and simple games of chance.	# j) Interpret and apply probability concepts to practical situations, including odds of success or failure in simple lotteries or games of chance.
		k) * Use the binomial theorem to solve problems.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).
# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Algebra

Algebra began in the use of systematic methods for solving problems and numerical puzzles by mathematicians in the Middle East, South Asia, and China, and made its way to Europe in the late Middle Ages. The modern symbolic notation, with letters to stand for unknowns and constants, was developed in the 16th century. The notation so greatly enhanced the power of the algebraic method that the basic ideas of both analytic geometry and calculus were developed within a century.

The increased use of algebra led to study of its formal structure. Gradually, the "rules of algebra" were distilled into a compact summary of the principles behind algebraic manipulation. In the 19th century, these principles (e.g., commutativity, distributivity) were codified into a deductive system parallel to that of Euclidean geometry. A corresponding line of thought produced a simple but flexible concept of function and also led to the development of set theory as a comprehensive background for mathematics. When taken broadly as including these ideas, the study and uses of algebra reach from the foundations of mathematics to the frontiers of current research.

The notion of variable—a symbol that can stand for any member of an identified set—has multiple facets (e.g., as an unknown, parameter, or varying quantity); variables are used in many ways in school mathematics. Variables are used to express structural generalizations such as the commutativity of addition. In formulas such as $d = rt$ or $c = \sqrt{a^2+b^2}$, variables stand for quantities that may take on a variety of values. In problem solving, a variable may represent an unknown quantity. The study of functions includes attention to independent variables, dependent variables and parameters.

When students make abstractions and generalizations about numbers and operations in early arithmetic by attending to underlying structure, they are engaging in algebraic thinking even though the formalism of algebraic notation may not be evident. As students progress through the grades, they continue to engage in algebraic thinking and they add more algebraic formalism to their repertoire.

By grade 4, students are expected to recognize and extend simple numeric patterns as a foundation for a later understanding of function. They begin to understand the meaning of equality and some of its properties, as well as the idea of an as-yet-unknown quantity as a precursor to the concept of variable. They also begin to informally explore properties of operations, including how inverse operations can be used to simplify a computation or how numbers can be decomposed and recomposed for more efficient computational strategies.

As students move into grade 8, the ideas of variable, covariation (two or more quantities varying simultaneously), and function become more important. By using variables to describe patterns and solve simple equations, students become familiar with manipulating them. Representations of covariation in tables, verbal descriptions, symbolic descriptions, and graphs can combine to promote a flexible grasp of the idea of function. Linear functions receive special attention: they connect to the ideas of proportionality, ratio, and rate, forming a bridge that will eventually link arithmetic to calculus. Symbolic manipulation in the relatively simple context of linear equations is reinforced by other ways of finding solutions, including graphing by hand or with technology.

By grade 12, students are expected to be skillful at manipulating and interpreting more complex expressions. Nonlinear functions, especially quadratic, power, and exponential functions whose graphs are accessible using graphing technology, are used by students to solve real-world problems. Grade 12 students are also expected to be accomplished at translating verbal descriptions of problem situations into symbolic form. Also, by grade 12, students should understand expressions involving several variables, systems of linear equations, and solutions to inequalities. The 2026 Algebra objectives are shown in Exhibit 2.6.

Exhibit 2.6. Algebra (Alg)

Alg – 1. Patterns, relations, and functions		
Grade 4	**Grade 8**	**Grade 12**
a) Recognize, describe (in words or symbols), or extend simple numerical and visual patterns.	a) Recognize, describe, or extend numerical and visual patterns using tables, graphs, words, or symbols.	a) Recognize, describe, or extend numerical patterns, including arithmetic and geometric sequences (progressions).
		b) Express linear and exponential functions in recursive and explicit form given a verbal description, table, or some terms of a sequence.
c) Given a description, extend or find a missing term in a pattern or sequence.	c) Examine or create patterns, sequences, or linear functions expressed as a rule numerically, verbally, or symbolically.	
d) Create a different representation of a pattern or sequence given a verbal description.		
	e) Identify functions as linear or nonlinear or contrast distinguishing properties of functions from tables, graphs, or equations.	e) Identify or analyze distinguishing properties of linear, quadratic, rational, exponential, or *trigonometric functions from tables, graphs, or equations.
	f) Interpret the meaning of slope or intercepts, or determine the rate of change between two points on a graph of a linear function.	
		g) Determine whether a relation, given in verbal, symbolic, tabular, or graphical form, is a function.
		h) Recognize and analyze the general forms of linear, quadratic, rational, exponential, or *trigonometric functions.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Exhibit 2.6. Algebra (continued)

Alg – 1. Patterns, relations, and functions (continued)		
Grade 4	**Grade 8**	**Grade 12**
		i) Determine the domain and range of functions given in various forms and contexts.
		j) * Given a function, determine its inverse if it exists and explain the contextual meaning of the inverse for a given situation.

Alg – 2. Algebraic representations		
Grade 4	**Grade 8**	**Grade 12**
a) Translate between different representational forms (symbolic, numerical, verbal, or pictorial) of whole number relationships (such as from a written description to an equation or from a function table to a written description).	a) Translate between different representations of linear expressions using symbols, graphs, tables, diagrams, or written descriptions.	a) Create and translate between different representations of algebraic expressions, equations, and inequalities (e.g., linear, quadratic, exponential, or *trigonometric) using symbols, graphs, tables, diagrams, or written descriptions.
	b) Interpret and compare representations of linear relationships expressed in symbols, graphs, tables, diagrams, or written descriptions.	# b) Interpret and compare representations of relationships expressed in symbols, graphs, tables, diagrams (including Venn diagrams), or written descriptions.
	c) Graph or interpret points represented by ordered pairs of numbers on a rectangular coordinate system.	
	d) Solve problems involving coordinate pairs on the rectangular coordinate system.	d) Perform or interpret transformations on the graphs of linear, quadratic, exponential, and *trigonometric functions.
		e) Make inferences or predictions using an algebraic model of a situation.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.6. Algebra (continued)

Alg – 2. Algebraic representations (continued)		
Grade 4	**Grade 8**	**Grade 12**
	f) Identify or represent functional relationships in meaningful contexts including proportional, linear, and common nonlinear relationships (e.g., compound interest, bacterial growth) in tables, graphs, words, or symbols.	# f) Given a real-world situation, determine if a linear, quadratic, rational, exponential, *logarithmic, or *trigonometric function fits the situation.
		# g) Solve problems involving exponential growth and decay.
		h) * Identify distinguishing characteristics of exponential, logarithmic, and rational functions (e.g., discontinuity, asymptotes, concavity).

Alg – 3. Variables, expressions, and operations		
Grade 4	**Grade 8**	**Grade 12**
a) Use letters and symbols to represent an unknown quantity in a simple mathematical expression.		
b) Express simple mathematical relationships using expressions, equations, or inequalities.	b) Write algebraic expressions, equations, or inequalities to represent a situation.	b) Write algebraic expressions, equations, or inequalities to represent a situation.
	c) Perform basic operations, using appropriate tools, on linear algebraic expressions (including grouping and order of multiple operations involving basic operations, exponents, roots, simplifying, and expanding).	c) Perform basic operations, using appropriate tools, on algebraic expressions including polynomial and rational expressions.
		d) Write equivalent forms of algebraic expressions, equations, or inequalities to represent and explain mathematical relationships.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).
# Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.6. Algebra (continued)

Alg – 3. Variables, expressions, and operations (continued)		
Grade 4	**Grade 8**	**Grade 12**
		# e) Evaluate algebraic expressions, including polynomials and rational expressions.
		f) Use function notation to evaluate a function at a specified point in its domain and combine functions by addition, subtraction, multiplication, division, and composition.
		g) * Determine the sum of finite and infinite arithmetic and geometric series.
		h) Use basic properties of exponents and *logarithms to solve problems.

Alg – 4. Equations and inequalities		
Grade 4	**Grade 8**	**Grade 12**
a) Find the unknown(s) in a whole number sentence (e.g., in an equation or simple inequality like [_] + 3 > 7).	a) Solve linear equations or inequalities (e.g., Solve for x in $ax + b = c$ or $ax + b = cx + d$ or $ax + b > c$).	a) Solve linear, rational, or quadratic equations or inequalities, including those involving absolute value.
b) Interpret "=" as an equivalence between two values and use this interpretation to solve problems.		b) * Determine the role of hypotheses, logical implications, and conclusions in algebraic arguments about equality and inequality.
c) Verify a conclusion using simple algebraic properties derived from work with numbers (e.g., commutativity, properties of 0 and 1).	c) Make, validate, and justify conclusions and generalizations about linear relationships.	c) Use algebraic properties to develop a valid mathematical argument.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Exhibit 2.6. Algebra (continued)

Alg – 4. Equations and inequalities (continued)		
Grade 4	**Grade 8**	**Grade 12**
	d) Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically (e.g., $ax + b = c$ or $ax + b = cx + d$).	# d) Analyze situations, develop mathematical models, or solve problems using linear, quadratic, exponential, or *logarithmic equations or inequalities symbolically or graphically.
	e) Interpret relationships between symbolic linear expressions and graphs of lines by identifying and computing slope and intercepts (e.g., in $y = ax + b$, know that a is the rate of change and b is the vertical intercept).	e) Solve (symbolically or graphically) a system of equations or inequalities and recognize the relationship between the analytical solution and graphical solution.
	f) Use and evaluate common formulas (e.g., relationship between a circle's circumference and diameter, $C = \pi d$, distance and time under constant speed).	# f) Solve problems involving special formulas such as: $A = P(I + r)^t$ or $A = Pe^{rt}$.
		# g) Solve an equation or formula involving several variables for one variable in terms of the others.
		h) * Solve quadratic equations with complex roots.

* Objectives that describe mathematics content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra with statistics).

Grade 12 objectives that provide opportunities for questions in mathematical literacy.

Revisions of the 2017 Content Objectives

Revisions to the 2017 NAEP mathematics content objectives resulted from consideration of a wide range of relevant sources. These included research on mathematical development and learning, each state's standards and frameworks for mathematics instruction and assessment in the United States, reviews of state standards in comparison to NAEP objectives (e.g., Johnston et al., 2018), research on the alignment between NAEP items and common standards (e.g., Daro, Hughes, & Stancavage, 2015), policy statements informing state standards (e.g., NCTM, 2000, 2014, 2018; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), *Guidelines for Assessment and Instruction in Statistics Education* (GAISE; Bargagliotti et al., 2020), *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME; Garfunkel & Montgomery, 2019), the content of leading international assessments (e.g., PISA [OECD, 2019] and TIMSS [NCES, 2019]), the professional judgment and experience of Panel members, and feedback obtained from readers of draft versions of this framework.

Though overlapping, these sources were not in complete agreement regarding the mathematics students need to know and be able to do. Using this range of sources resulted in a set of objectives that cannot and will not be representative of what every child in the U.S. is taught by a given grade, nor will they conform precisely to the stated achievement objectives of any single state or professional organization. At the same time, the resulting objectives are tightly linked to acknowledged aspirations for the mathematics U.S. students should have an opportunity to learn. The content delineated here focuses on mathematical ideas that students are likely to have encountered in school.

Revisions attended to both current state standards—where the nation is now—and where the nation is likely headed. Updates to the content objectives were also motivated by several other considerations, including precision and accuracy of the language used to describe an objective; developmental appropriateness of objectives at a particular grade level, based on current research and state policies; and shifts in content emphases since the last framework update. In the case of a limited number of objectives that are not common in the majority of U.S. state standards, guidance came from the ways leading states and nations situate those topics in their respective content objectives.

Restructuring of "Mathematical Reasoning" as a Subtopic

Mathematical Reasoning subtopics appeared in the previous NAEP Mathematics Assessment Framework (Governing Board, 2017a) in Number Properties and Operations, Geometry, Data Analysis, Statistics, and Probability, and Algebra. With the introduction of the NAEP Mathematical Practices (see Chapter 3), most of the Mathematical Reasoning objectives will be measured by items aligned to a content objective and classified with one of the NAEP Mathematical Practices. To preserve attention to content that was uniquely present in some of the Mathematical Reasoning objectives, some content from those objectives was incorporated into other subtopics' objectives (e.g., Number and Operations subtopic 3.e in grades 4 and 8 was "Interpret…" and is now "Interpret, explain, or justify…"; for more details on how these changes affect item development, see the *Assessment and Item Specifications* document).

Changes at Grade 4

In the early grades, up through grade 4, there is a distinction between NAEP content area arrangement and the arrangement common in many states' assessment standards. Most state assessments use three to five areas in the early grades, but these do not parallel the five areas used in NAEP. At the same time, it must be noted that analysis of state standards has indicated that some content in the previous objectives is now not regularly part of U.S. schooling until grade 5 or later (Daro et al., 2015; Hughes, Daro, Holtzman, & Middleton, 2013; Johnston et al., 2018). To address this, some objectives were removed at grade 4. In many cases, grade 8 objectives were similar and more appropriately timed to assess students on mathematics they would have had a chance to learn. Additionally, research comparing states' standards for curriculum and instruction with NAEP assessment objectives suggested that some content commonly taught by grade 4 was absent from NAEP (Johnston et al., 2018). Careful review of this analysis led to the modification or addition of objectives at grade 4. Research and development on the use of the equal sign as an equivalence between two values and its importance in the foundation for algebraic thinking (Carpenter, Franke, & Levi, 2003) has meant states include more attention to it. This greater attention led to the addition of one related objective in grade 4 Algebra. Increased work with certain concepts in early grades since the last NAEP Mathematics Framework update led to one addition and several modifications of grade 4 Number Properties and Operations objectives. Similarly, several grade 4 objectives in Data Analysis, Statistics, and Probability were modified to reflect current language use for noticing, using, and interpreting data.

Changes at Grade 8

Since the last NAEP framework update, there have been shifts in state standards in expectations about understanding and use of rates, recognition of pattern, and greater attention to data, statistics, and probability in grades 5, 6, 7, and 8 (i.e., after grade 4; Johnston et al., 2018). As a result, the grade 8 objectives in Data, Statistics, and Probability were revised to clarify expectations, and three grade 8 objectives were deleted because similar grade 4 objectives or grade 12 objectives were more appropriately timed to assess what students have an opportunity to learn.

Changes at Grade 12

At grade 12, as in the other grades, descriptions of objectives were edited to clarify measurement intent. Added in grade 12 were two objectives in Geometry and Measurement: one about periodicity of functions and one on applying geometric properties among similar figures in two and three dimensions. In some cases where an objective was identified as beyond what is commonly taught in grade 12, an asterisk (*) was added. Also, to support the possible reporting of Mathematical Literacy as a particular way in which students know and do mathematics at grade 12, a number sign (#) was added to indicate objectives relevant to the exploration of this reporting.

Changes in Item Distribution

As previously noted, the last decade has seen a shift of data and related topics to grades 5, 6, 7, and 8. Hence, the proportion of items for Data Analysis, Statistics, and Probability went up for grade 8 (from 15% to 20%) and down for grade 4 (from 10% to 5%). Concurrently, greater attention to fractions in grade 4 across states led to an increase in the proportion of Number

Properties and Operations items (from 40% to 45%). Measurement in contexts that are not geometric play a smaller role in grade 8 than geometry topics, and the proportion of such items was reduced (from 15% to 10%). By grade 12, most new measurement ideas are in geometric contexts and, as in the previous NAEP Mathematics Framework, measurement and geometry continue to be treated together in the item distribution for grade 12. In fact, the distribution of items for each content area at grade 12 remains the same, reflecting the delineation of essential concepts in the literature on high school learning (NCTM, 2018).

Chapter 3

NAEP Mathematical Practices

Interest in students' mathematical practices has been growing for over 40 years. Seminal work by authors such as Collins and Stevens (1983), Lave (1988), Saxe (1988), and Schoenfeld (1985) focused on the cognitive skills and strategies used by mathematics experts and adults "in the wild" (i.e., outside of school). This line of research led to a distillation of the specific behaviors engaged during mathematical reasoning and problem solving, illuminating what are now called "practices" of mathematics.

Mathematics education research has also experienced a "social turn" (Lerman, 2000), marked by a shift toward investigating mathematics learning as it is situated in social activity, including discourse practices (Adler, 1999; Bell & Pape, 2012; Black, 2004; Civil & Planas, 2004; Enyedy, 2003; Ernest, 1998; Moschkovich, 2007, 2008; NCTM, 1991; van Oers, 2001). Students use their mathematical knowledge and skill in the social settings of school and home, on the basketball court, or in games they play with friends. The 2026 NAEP Mathematics Framework captures this broader and more complete picture of what it means to know and do mathematics. For the first time, NAEP Mathematics includes mathematical practices as a fundamental component of the assessment (see Exhibit 3.1). This chapter offers a brief overview of the research literature on mathematical practices as a whole and describes these five key NAEP Mathematical Practices in depth. As was the case with the content areas in Chapter 2, these five areas are not meant to be inclusive of all possible mathematical activity.

Exhibit 3.1. Summary of NAEP Mathematical Practices

NAEP Mathematical Practice 1: Representing

Recognizing, using, creating, interpreting, or translating among representations appropriate for the grade level and the mathematics being assessed.

NAEP Mathematical Practice 2: Abstracting and Generalizing

Decontextualizing, identifying commonality across cases, items, problems, or representations, and extending one's reasoning to a broader domain appropriate for the grade level and the mathematics being assessed.

NAEP Mathematical Practice 3: Justifying and Proving

Creating, evaluating, showing, or refuting mathematical claims in developmentally and mathematically appropriate ways.

NAEP Mathematical Practice 4: Mathematical Modeling

Making sense of a scenario, identifying a problem to be solved, mathematizing it, applying the mathematization to reach a solution, and checking the viability of the solution in developmentally and mathematically appropriate ways.

NAEP Mathematical Practice 5: Collaborative Mathematics

The social enterprise of doing mathematics with others through discussion and collaborative problem solving whereby ideas are offered, debated, connected, and built-upon toward solution and shared understanding. Collaborative mathematics involves joint thinking among individuals toward the construction of a problem solution in developmentally and mathematically appropriate ways.

Selecting Mathematical Practices for NAEP

The five NAEP Mathematical Practices are a particular distillation—for the purposes of assessment—of more than 40 years of research and development. They reflect a review of current scholarship, national and international assessment frameworks, national standards, and state standards more broadly.

To understand what mathematical practices are, it may be helpful to consider what they are not. Although practices underlie and contribute to mathematical reasoning, they are not completely synonymous with it, because many other skills contribute to mathematical reasoning, such as working memory (Geary, Hoard, Byrd-Craven, & DeSoto, 2004) and computational fluency (Geary, Liu, Chen, Saults, & Hoard, 1999). Similarly, although mathematical practices may contribute to conceptual understanding, the two are not interchangeable. On some accounts, conceptual understanding is knowledge of the underlying structure and relations represented in mathematics that transcends application of familiar algorithms (Eisenhart et al., 1993; Hiebert & Lefevre, 1986). In contrast, practices are fluid and responsive to both familiar and unfamiliar problems. Indeed, it is just as likely that conceptual understanding improves students' mathematical practices as it is that practices themselves improve conceptual understanding.

An increasing emphasis on mathematical practices is evident in state and national standards (NCTM, 1991, 2000, 2014). It is now generally agreed that knowing and doing mathematics entail engaging in practices such as generalizing, conjecturing, justifying, mathematizing, solving problems, communicating, and sense-making (Barbosa, 2006; Goos, 2004; Goos, Galbraith, & Renshaw, 2002; Hufferd-Ackles, Fuson, & Sherin, 2004; Hussain, Monaghan, & Threlfall, 2013; Lau, Singh, & Hwa, 2009; Truxaw & DeFranco, 2008). As students grapple with and discuss mathematical ideas and problems—individually and together—they engage in such mathematical practices, which serve to familiarize them with the norms of doing mathematics (Herbel-Eisenmann & Cirillo, 2009). The inclusion of NAEP Mathematical Practices is not separate from the mathematics content of Chapter 2. These practices are described separately to indicate the significant change to the NAEP Mathematics Framework in sufficient detail.

The term "mathematical practices" has been used by the field in a variety of ways, with state standards and NCTM standards offering two widely disseminated descriptions. Five specific practices have been selected for emphasis on the 2026 NAEP Mathematics Assessment; these are referred to throughout this framework as the NAEP Mathematical Practices. As further detailed in Chapter 4, the assessment is designed to measure content and practices together. However, not all items will include an assessed NAEP Mathematical Practice. In fact, not all NAEP content objectives need to be assessed alongside a NAEP Mathematical Practice. Some items will continue to assess content outside of the particular NAEP Mathematical Practices, such as items that focus on algorithms, procedural fluency, precision, tool use, or mathematical practices other than the five that are the focus for the NAEP Mathematics Assessment.

There are commonalities across the NAEP Mathematical Practices and the practices described in policy documents and common in state standards. For example, the NAEP Mathematical Practices and the NCTM Mathematical Process Standards include communication and collaboration, while communication is a subtext in several of the mathematical practices common in state standards (e.g., in critiquing the reasoning of others). Representing in the doing, teaching, and learning of mathematics is a process standard in NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), *Principles and Standards for School Mathematics* (2000), and *Catalyzing Change in High School Mathematics* (2018) and is also a NAEP Mathematical Practice. The NCTM Process Standards include reasoning and proof, and states' standards for mathematical practice include constructing viable arguments; both are similar to the NAEP Mathematical Practice of Justifying and Proving. The NAEP Mathematical Practice of Abstracting and Generalizing is similar to a common state standard for mathematical practice about reasoning abstractly and quantitatively. Mathematical Modeling is in most states' standards for mathematical practice as well as a NAEP Mathematical Practice.

Operationalizing the NAEP Mathematical Practices

A description of each NAEP Mathematical Practice follows. Although each practice is treated as distinct, they are interrelated with one another and with content, as is demonstrated in the examples provided throughout. In designing NAEP items, it may be impossible to completely isolate a particular mathematical practice in an item. When items assess multiple aspects of mathematics, it should be possible to identify a primary content focus and a primary practice focus. The former has been done on NAEP Mathematics Assessments for many years, and the latter should be possible moving forward. Further, the practices fundamentally intersect with, and develop in relation to, content. In this sense, the practices cut across grade levels, as well as across *NAEP Basic*, *NAEP Proficient*, and *NAEP Advanced* achievement levels. This approach to mathematical practices is reflected in policy and state standards, where mathematical content standards are offered and described by grade levels, while practices cut across grade levels. Just as some mathematics content objectives are more likely to interact with others in items, some mathematical practices are more likely to be found in connection with certain mathematics objectives. At the end of this chapter, Exhibits 3.25A–3.25C provide examples of where and how the five NAEP Mathematical Practices might be assessed within the NAEP mathematics content areas at each grade level. The tables are illustrative, not exhaustive, of ways practices could be assessed within content areas.

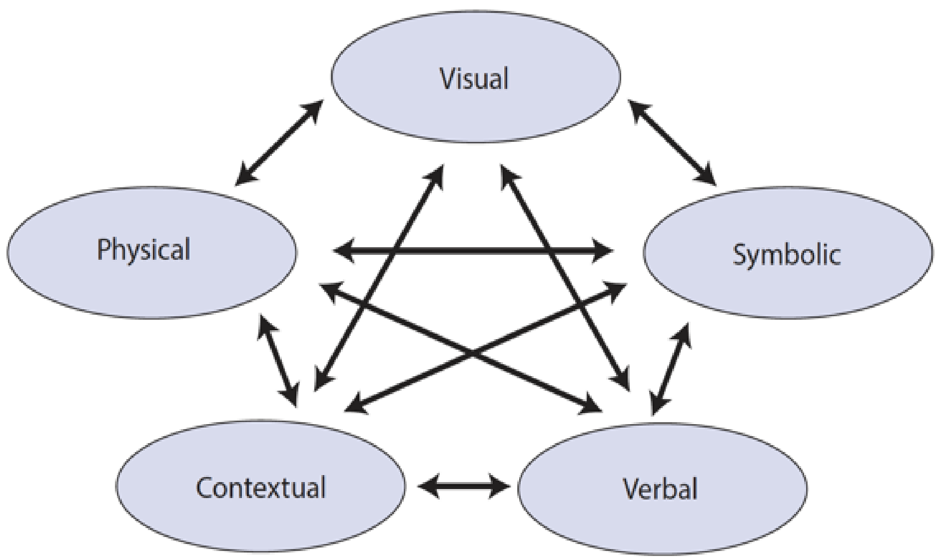
All released NAEP items used as exhibits in this framework were accessed using the online NAEP Questions Tool (NCES, n.d.). Some examples are from other sources, including example items from the Smarter Balanced Assessment Consortium (SBAC), and adaptations of tasks from policy and curriculum documents. The source for each item is cited in related text description about the item.

NAEP Mathematical Practice 1: Representing

> *Representing: Recognizing, using, creating, interpreting, or translating among representations appropriate for the grade level and the mathematics being assessed.*

Representing mathematical ideas and using mathematical representations to make sense of and solve problems is central to mathematics. Students create representations themselves, or in collaboration with other students, and they reason from or translate between standard representations (e.g., graphs, tables, geometric drawings) (Lesh, Post, & Behr, 1987; NCTM, 2014). Tripathi (2008) argues that variety in representations "is like examining a concept through a variety of lenses, with each lens providing a different perspective that makes the picture (concept) richer and deeper" (p. 439). Exhibit 3.2, from *Principles to Actions* (NCTM, 2014, p. 25) illustrates some of the types of representation and the relationships among them.

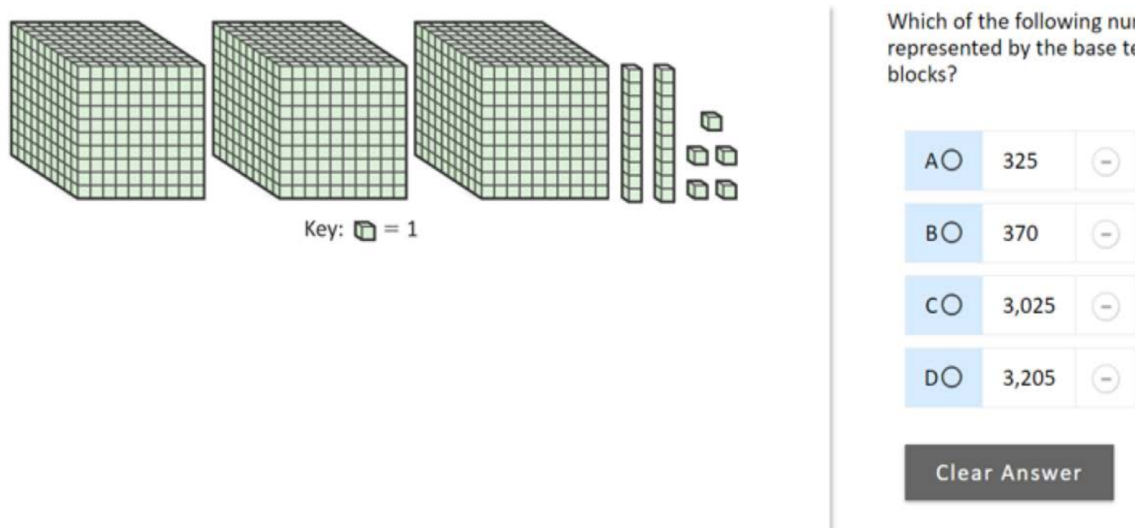
Exhibit 3.2. Types and Connections Among Mathematical Representations



According to the National Research Council (NRC, 2009), students, especially young ones, benefit from using physical objects or acting out processes during problem solving. Base 10 blocks (or blocks/tiles representing other bases), fraction strips/bars, red–black integer tiles, and algebra tiles are all examples of physical representations of number and operation that are used to enhance students' understanding of concepts in elementary and middle grades. These visual and physical representations connect, eventually, to symbolic representations as well. Visual representations also play a particularly powerful role in helping students make sense of problems and understand mathematical concepts and procedures. For instance, arrays of squares in a grid can be used to represent area models for mathematical operations such as multiplication and division in early elementary grades, then later for multiplication of algebraic expressions. Additionally, students create, use, and reason about multiple representations for a given mathematical idea or relationship in contextually relevant ways.

The grade 4 item in Exhibit 3.3, from the 2017 NAEP Mathematics Assessment, provides an image of base 10 blocks and asks students to determine the number shown. In answering the question, students connect a visual representation of a number to its symbolic representation in base 10. The item is framed to elicit a basic level response, whereas a revision of the question to a constructed-response format—such as “How many unit cubes are there in all?”—might give students an opportunity to demonstrate skill in interpreting a visual representation.

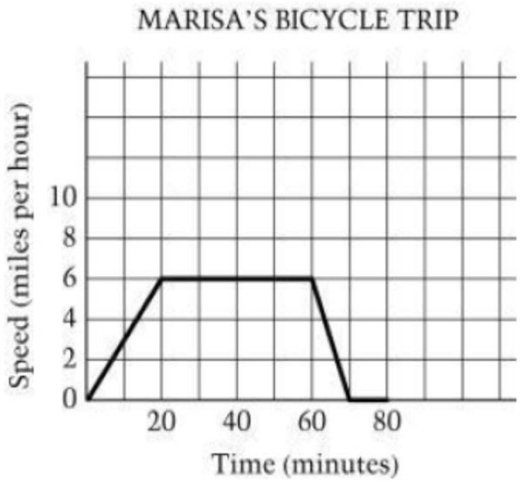
Exhibit 3.3. Grade 4 NAEP Number Sense Example: Interpreting a Visual Representation



The grade 8 item in Exhibit 3.4, from the 2003 NAEP Mathematics Assessment, demonstrates how students might provide a verbal representation from a graphical representation, or generate several alternative representations based on a problem situation. The item asks a student to take a graphical representation and work backward to a context that could fit that representation.

Alternatively, students could be asked to create their own graphical representation of a bicycle trip over time from a given verbal description of a trip. More realistic graphs of trips could be presented; for example, the item might offer a graph of a bicycle trip with more of a range and variety of speeds, including where the speed is zero at times mid-trip. Students could be given several different explanations that were provided by hypothetical students and asked to decide if those explanations correctly match the representation in the graph, or what an alternative explanation might be.

Exhibit 3.4. Grade 8 (and/or Grade 12) NAEP *Bicycle Trip* Item



The graph above represents Marisa's riding speed throughout her 80-minute bicycle trip. Use the information in the graph to describe what could have happened on the trip, including her speed throughout the trip.

During the first 20 minutes, Marisa

From 20 minutes to 60 minutes Marisa

From 60 minutes to 80 minutes Marisa

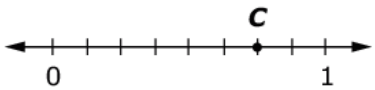
The item in Exhibit 3.5, from the Smarter Balanced Assessment Consortium (SBAC), provides a point on a number line that represents a distance, along with additional written information. As they work to solve the problem, students are expected to engage with the measurement represented on the number line in conjunction with some additional information, recognize the representation of a fraction, and apply it within the given context.

Similarly, the SBAC item in Exhibit 3.6 asks students about two more ways of representing. In it, students select the written statement that could be represented by the given equation, connecting a context to a symbolic representation.

Exhibit 3.5. Grade 5 SBAC Item (NAEP 2026 Objective Grade 8 Num – 3.a)

3218

Chris and Ben walked home from school. The distance Chris walked, in miles, is represented by point *C* on the number line.



Ben walked $\frac{1}{4}$ mile less than Chris walked.

Enter the distance, in miles, Ben walked.

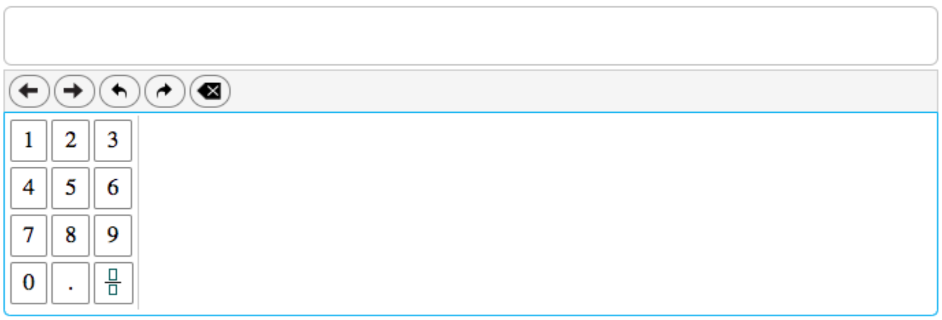


Exhibit 3.6. Grade 5 SBAC (NAEP 2026 Objective Grade 8 Num – 3.a)

3274

Which situation can be represented by this equation?

$4 \div \frac{1}{8} = \square$

Ⓐ Jack has 4 pieces of fabric. Each piece is $\frac{1}{8}$ of a yard long. How many yards of fabric does Jack have?

Ⓑ Jack has 4 pieces of fabric. He gets $\frac{1}{8}$ more yards of fabric. How many yards of fabric does Jack have now?

Ⓒ Jack has 4 yards of fabric. He gives away $\frac{1}{8}$ of his pieces of fabric. How many pieces of fabric does Jack have left?

Ⓓ Jack has 4 yards of fabric. He cuts the fabric into pieces $\frac{1}{8}$ of a yard long. How many pieces of fabric does Jack have?

NAEP Mathematical Practice 2: Abstracting and Generalizing

Abstracting and Generalizing: Decontextualizing, identifying commonality across cases, items, problems, or representations, and extending one's reasoning to a broader domain appropriate for the grade level and the mathematics being assessed.

Abstracting

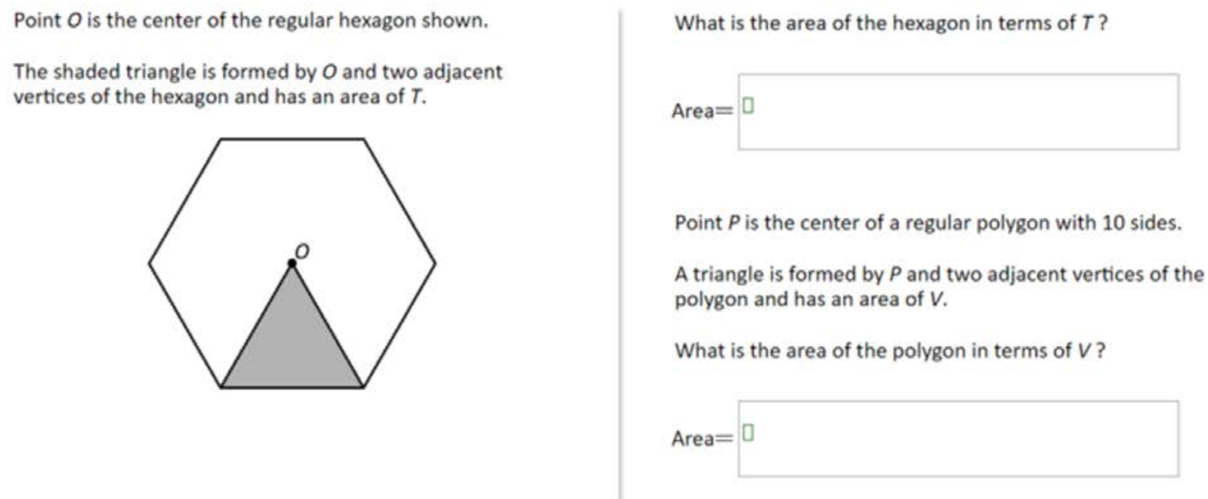
Students learning and doing mathematics also engage in the practice of abstracting and generalizing. An essential element of mathematical learning and problem solving is the ability to reason abstractly and to develop, test, and refine generalizations. In reasoning abstractly, students engage in the process of *decontextualizing*: Students abstract ideas in a given problem or context and express and manipulate them in a manner independent of their contextual references. Decontextualizing can foster an understanding of the relationships among problem contexts and written or symbolic forms, as well as an understanding of how mathematical expressions might be transformed to facilitate a solution strategy. Abstracting is also a critical activity for fostering generalizing; it enables a consideration of concepts and relationships decontextualized from specific examples or cases, which can support the formation of a more general rule or relationship.

Young students, for instance, can notice patterns of additive commutativity, such as 3 + 7 yielding the same sum as 7 + 3. In this instance, decontextualization would include finding a way to represent this relation independent of particular numbers, as a more general identity. Younger students might express this general identity verbally or with pictures, or with the use of a generic example. Older students might express this identity algebraically as $a + b = b + a$. Reasoning abstractly can also support recognizing similar mathematical structures across different problems or domains. For example, one could see the multiplication of two binomials $(2x + 7)(3x + 2)$ as a more general version of multiplying 27 by 32.

Consider the grade 8 Geometry item in Exhibit 3.7, from the 2017 NAEP Mathematics Assessment. This item requires students to express the area of the hexagon in terms of the area of the given shaded triangle. Students are then asked to extend their reasoning to a 10-sided figure. Thus, students are first challenged to reason structurally by mentally comparing the area of the triangle formed by the hexagon's center and two adjacent vertices with the area of the entire figure. Students are then further tasked with extending their reasoning from the specific case of the hexagon to another regular polygon.

Although a student could solve the problem in Exhibit 3.7 by drawing a 10-sided polygon and the specified triangle, and then counting the number of triangles that comprise the polygon, a student could also carry out this operation mentally rather than drawing it out. Also, the item could be revised to elicit decontextualizing beyond the hexagon, thinking about the relationship between the specified triangle and any regular polygon. In the later grades, students could be expected to express their reasoning algebraically and develop and prove a conjecture about the general relationship between the triangle and any n-sided regular polygon.

Exhibit 3.7. Grade 8 NAEP Geometry Item



Abstracting can occur across different domains. It can be addressed in reasoning about figures and their relationships in geometry, about number theory in number properties and operations, or about equivalence or functional relationships in algebra. How one decontextualizes or reasons with structure will differ across the domains, but these are processes students can employ in all five content areas included in the NAEP Mathematics Assessment.

Generalizing

Mathematics education researchers and policymakers have defined generalizing in a number of ways. Historically, generalization has been defined as an individual, cognitive construct (e.g., Carraher, Martinez, & Schliemann, 2008), where generalization is the act of identifying a property that holds for a larger set of mathematical objects or conditions than the number of individually verified cases. For instance, Harel and Tall (1991) described generalization as the process of “applying a given argument in a broader context” (p. 38), and Radford (2007) argued that generalization involves identifying a commonality based on particulars and then extending it to all terms.

More recently, researchers have begun to address generalizing as a construct that is both social and cognitive; that is, it can occur either individually or collectively. Therefore, for NAEP, generalizing is an individual or collective practice of (a) identifying commonality across cases, (b) extending reasoning beyond the domain in which it originated, and/or (c) deriving broader results from particular cases (Ellis, 2007). Its social dimensions make it relevant to the NAEP Collaborative Mathematics practice.

Several aspects of mathematical reasoning can foster generalizing. As previously mentioned, abstracting and decontextualizing are important mental actions that support generalizing. Other actions that support generalizing include visualizing, focusing, reflecting, connecting, and expressing. *Visualizing* involves seeing patterns or structural relationships, as well as imagining a set of relationships beyond what is perceptually available. *Focusing* is attending to particular details, characteristics, properties, or relationships above others. This can include examining a particular case in a pattern or attending to figural or numerical cues. *Reflecting* involves actions such as thinking back on the operations one has carried out, observing one's method in solving problems, or examining the rules that govern a given pattern. *Connecting* is the identification of relationships among tasks, representations, or properties. Making connections between representations or identifying and operating on structural similarities can foster the development of generalizations. Finally, *expressing* involves depicting a generalization verbally or in writing. Describing generalizations in words can support the subsequent development of algebraically represented generalizations.

Like abstracting, generalizing can occur across the content areas and grade bands. Existing NAEP Mathematics Assessment items contain a number of generalization tasks in which students are asked to determine a rule guiding the pattern of number terms in a sequence. In some items, potential rules are provided for students who are prompted only to attend to the action required to move from one term in the sequence to the next. In other items, students must determine a rule themselves, such as for the grade 12 item in Exhibit 3.8. It is worth noting that for items such as the one in Exhibit 3.8, there could be any number of non-equivalent rules to describe the pattern, so it may be more appropriate to ask students to provide "a" rule rather than "the" rule. Notice that for part c of this grade 12 item, students are expected to write a formal algebraic rule for moving from the n^{th} term to the $(n + 1)^{\text{st}}$ term of Sequence I by identifying an explicit rule for the n^{th} term of Sequence II. In other items, students may be tasked with determining a recursive, rather than explicit, rule to find the n^{th} term in a sequence.

Exhibit 3.8. Grade 12 NAEP Number Pattern Item

Sequence I: 3, 5, 9, 17, 33, . . .

Sequence I, shown above, is an increasing sequence. Each term in the sequence is greater than the previous term.

a. Make a list of numbers that consists of the positive differences between each pair of adjacent terms in Sequence I. Label the list Sequence II.

b. If this same pattern of differences continues for the terms in Sequence I, what are the next two terms after 33 in Sequence I ?

6th term ______________________

7th term ______________________

c. Write an algebraic expression (rule) that can be used to determine the n^{th} term of Sequence II, which is the difference between the $(n+1)^{\text{st}}$ term and the n^{th} term of Sequence I.

Students can also be challenged to engage in the processes of generalizing in items that do not rely on pattern sequences, as in Exhibit 3.9. This item could support a number of possible generalizing processes, as well as the opportunity for abstracting. For instance, one could consider that for each coin (nickel, dime, quarter), there are two possible outcomes, H or T. Thus, a student could either systematically list outcomes to determine that there are 8 total outcomes or begin to think structurally to reason that for three coins and two outcomes per coin, there must be $2^3 = 8$ total outcomes. Alternatively, through systematic listing, a student could determine that there are $1 + 3 + 3 + 1$ outcomes, corresponding to 1 outcome with exactly zero Ts, 3 outcomes with exactly one T, 3 outcomes with exactly two Ts, and 1 outcome with exactly three Ts. Extending to the 4-coin case, for instance, students might determine that the number of outcomes is $1 + 4 + 6 + 4 + 1$, corresponding to 1 outcome with exactly zero Ts, 4 outcomes with exactly 1 T, 6 outcomes with exactly 2 Ts, 4 outcomes with exactly three Ts, and 1 outcome with exactly four Ts (and symmetrically but opposite for the number of Hs).

Exhibit 3.9. Grade 8 and/or Grade 12 Task (Adapted from a Grade 8 NAEP Item)

Three students each have a coin, one has a nickel, one has a dime, and the third student has a quarter. They flip their coins at the same time. Each coin can land either heads up (H) or tails up (T). List all the different possible outcomes for how the coins could land in the chart below. The list has been started for you.

Nickel	Dime	Quarter
H	H	H
H	H	T

What if a 4th student joins the group with a half-dollar coin? How many different ways could the 4 coins land? What if a 5th student joined with a penny–how many different ways could the 5 coins land?

One aspect of generalizing is identifying commonality across cases. Students might notice that the outcomes for the 3-coin and 4-coin cases can be structured according to the rows in Pascal's triangle. Or, students might reason that, like the 3-coin case, each of the positions in the 4-coin case has two possible outcomes, H or T, and thus the total number of possible outcomes must be $2^4 = 16$, and, more generally, for n coins, 2^n. An item like the one in Exhibit 3.9 affords a number of rich generalizing opportunities, regardless of whether students are expected to recognize that 2^n is the sum of the coefficients of the binomial expression $(a + b)^n$ (e.g., $2^4 = 1 + 4 + 6 + 4 + 1$).

NAEP Mathematical Practice 3: Justifying and Proving

Justifying and Proving: Creating, evaluating, showing, or refuting mathematical claims in developmentally and mathematically appropriate ways.

Justifying and proving are essential in all content areas and grade levels. Traditionally, proof was viewed as a form of mathematical argumentation pertaining first to high school geometry and not visited again until pre-calculus courses with proofs of trigonometric identities and proofs by mathematical induction. However, this changed in the last quarter of the 20th century. The *Principles and Standards for School Mathematics* emphasized the importance of justifying and proving at all levels of mathematics, noting that “reasoning and proof should be a consistent part of students’ mathematical experience in prekindergarten through grade 12” (NCTM, 2000, p. 56). Similarly, state standards highlight the activities students engage in as they learn to create valid mathematical arguments: making and investigating conjectures, developing particular forms of argument (e.g., deductive), and using a variety of proof methods (e.g., direct, counterexample). These are all considered components of the practice of justifying and proving.

Mathematical justification includes creating arguments, explaining why conjectures must be true or demonstrating that they are false, exploring special cases or searching for counterexamples, understanding the role of definitions and counterexamples, and evaluating arguments (Ellis, Bieda, & Knuth, 2012). A valid justification should show why a statement or conjecture is true or not true generally (i.e., for all cases) and, especially by grades 8 and 12, should do so by providing a logical sequence of statements, each building on already established statements, ideas, or relationships.

A justification is *not* based on authority, perception, popular consensus, or examples alone. As students engage in justifying, they may be tempted to rely on external sources to verify their ideas, such as their teacher or a textbook (Harel & Sowder, 1998). Students may also want to use examples to support their claims, concluding that a conjecture must be true because it holds for several different cases. Examples can and do play an important role in justifying and proving, particularly in terms of helping students make sense of statements, gain a sense of conviction, or revealing an underlying structure that could lead to a proof. But they do not suffice as a mathematical justification or proof except for proofs by exhaustion or counterexample.

A proof can have many different forms, including narrative, pictorial, diagram, two-column, or algebraic forms. The form used to represent a mathematical proof is valid as long as it communicates the proof’s essential features, namely, that it contains logically connected mathematical statements that are based on valid definitions and theorems. For instance, consider the grade 4 item in Exhibit 3.10.

Exhibit 3.10. Grade 4 Number Properties and Operations Proof Item

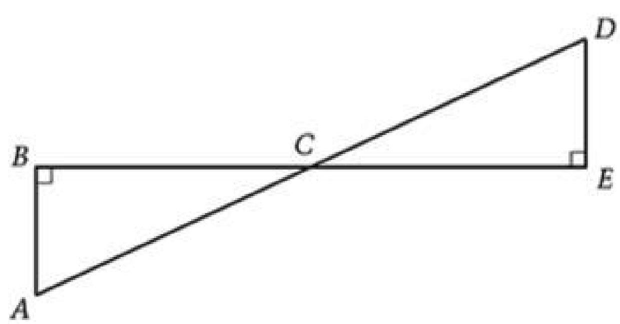
> Elise claims that if you multiply any whole number by 6, you will always get an even number for the answer. Provide an argument for why Elise is correct.

A grade 4 proof for the claim in Exhibit 3.10 could involve demonstrating with either pictures or symbols that the answer can always be separated into two equal parts, because 2 is a factor of 6, or that the answer can always be divided by 2 or cut in half because 2 already divides 6. An argument such as 6 x NUMBER = 3 x NUMBER + 3 x NUMBER might also be provided by fourth graders, demonstrating symbolically that the result can be split into two equal parts. Arguing from examples alone is not a justification, but in providing examples students may discover the key piece to demonstrate that 2 will always be a factor of the product.

A formal proof is a specific type of argument "consisting of logically rigorous deductions of conclusions from hypotheses" (NCTM, 2000, p. 55). In grade 12, students are expected to develop formal mathematical proofs. A proof uses definitions and theorems that are available without further justification, and a proof is valid only if the assumptions upon which it relies have already been shown to be true.

Often, the phrase "mathematical proof" conjures an image of the traditional two-column proof that is typical in high school geometry classrooms. This form of proof can be helpful for supporting students' efforts to develop a clear chain of statements, each relying on the prior, and for making sure that each statement is justified, as illustrated in Exhibit 3.11.

Exhibit 3.11. Grade 12 NAEP Geometry Proof Item



Given: C is the midpoint of $\overline{BE}$.
$\angle B$ and $\angle E$ are right angles.

Prove that $\overline{AC} \cong \overline{DC}$ and give a reason for each statement in your proof.

This item lends itself well to a two-column proof, particularly because it stipulates that a reason must be provided for each statement in the proof. One proof is as follows:

Statement	Reason
C is the midpoint of $\overline{BE}$	Given
$\angle B$ and $\angle E$ are right angles	Given
$\overline{BC} \cong \overline{EC}$	Definition of midpoint
$\angle B \cong \angle E$	Right angles are congruent
$\angle ACB \cong \angle DCE$	Vertical angles are congruent
$\triangle ACB \cong \triangle DCE$	Angle-Side-Angle (or Leg-Angle)
$\overline{AC} \cong \overline{DC}$	Corresponding parts of congruent triangles are congruent

Although this proof follows a typical form of school mathematics proof, there is nothing about the prompt that stipulates that the proof must occur in a two-column format. A narrative form of the proof in answer to the item in Exhibit 3.11 could also be appropriate, as seen below:

> The measures of $\angle BCA$ and $\angle ECD$ are equal because vertical angles have the same measure. We also know that the measures of $\angle B$ and $\angle E$ are the same because they are both right angles. Since C is the midpoint of $\overline{BE}$, $\overline{BC} \cong \overline{EC}$. So, by the angle-side-angle rule, triangle ACB is congruent to triangle DCE. Therefore, $\overline{AC} \cong \overline{DC}$ because corresponding parts of congruent triangles are congruent.

In addition to the various formats one can use to develop or present proofs, there are other ways of mathematically proving, disproving, or justifying a mathematical answer. These include developing deductive arguments, finding counterexamples, proving by exhaustion (i.e., verifying every possible case), and employing mathematical induction. Often, it may be easier to use a particular mode of argumentation based on the nature of the claim.

The process of refuting—demonstrating that a statement is false—is a key element of justification because conjecturing can produce both true and false statements. Students must understand that a single counterexample disproves a conjectured generalization.

An example of the value of finding a counterexample can be seen in the grade 12 algebra item in Exhibit 3.12. Here, one could identify a value for x that is, for instance, less than 5 but not also greater than –3 (e.g., $x = -10$). That single counterexample is sufficient to show that Dave's claim cannot be correct because $x = -10$ does not satisfy the statement $-3 < x < 5$.

Exhibit 3.12. Grade 12 NAEP Algebra Counterexample Item

Question A: If x is a real number, what are all values of x for which $x > -3$ and $x < 5$?

Question B: If x is a real number, what are all values of x for which $x > -3$ or $x < 5$?

Barbara said that the answers to the two questions above are different.

Dave said that the answers to the two questions above are the same.

Which student is correct?

○ Barbara ○ Dave

Explain why this student is correct. You may use words, symbols, or graphs in your explanation.

The questions at the start of the item in Exhibit 3.12 could be altered to give a grade 8 item:

Question A: If x is a number, what are all values of x for which $x \geq -3$?
Question B: If x is a number, what are all values of x for which $x > -3$?

The rest of the item would remain the same.

Similarly, only one counterexample is needed in the grade 8 Number Properties and Operations item in Exhibit 3.13. Multiplying 6 by any real number less than 1 will yield a result less than 6, confirming Tracy's claim and refuting Pat's claim.

Exhibit 3.13. Grade 8 NAEP Number Properties and Operations Counterexample Item

Tracy said, "I can multiply 6 by another number and get an answer that is smaller than 6."

Pat said, "No, you can't. Multiplying 6 by another number always makes the answer 6 or larger."

Who is correct? Give a reason for your answer.

Understanding that a single counterexample undermines a general claim is an important but difficult aspect of justification. Learning to search for counterexamples and explaining why they are justifications is only one aspect of refutation. Attempting to prove that a conjecture is false can also lead to the development of new insights or ideas, as well as to the formation of different conjectures that can then be explored, refuted, or proved.

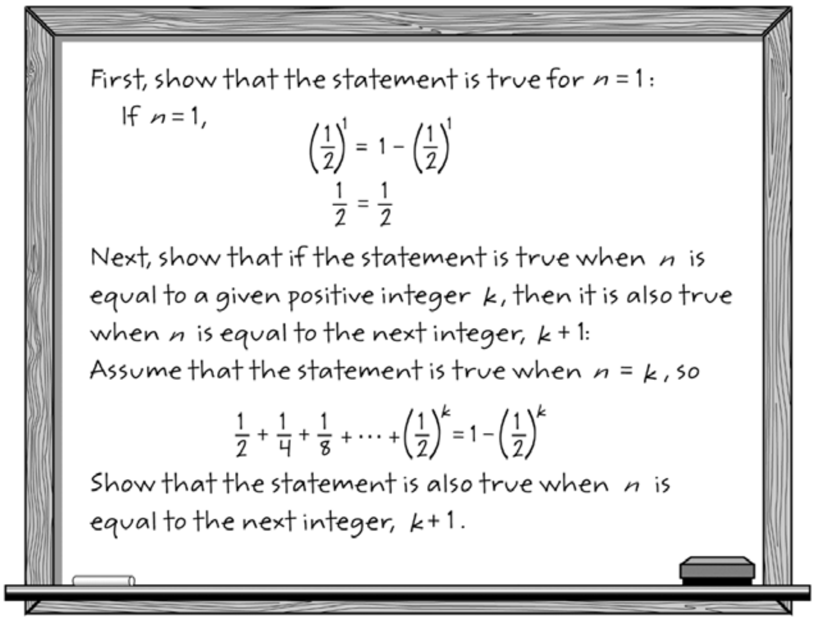
Some NAEP items require a specific mode of proof, such as the grade 12 Number Properties and Operations item in Exhibit 3.14.

Exhibit 3.14. Grade 12 NAEP Number Properties Mathematical Induction Item

A student was asked to use mathematical induction to prove the following statement.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\left(\frac{1}{2}\right)^n = 1-\left(\frac{1}{2}\right)^n$ for all positive integers n

The beginning of the student's proof is shown below.



Complete the student's proof by showing that if the statement is true when $n = k$, then it is also true when $n = k + 1$, where k is any positive integer.

Here, a student must use the tools of mathematical induction to complete the provided argument:

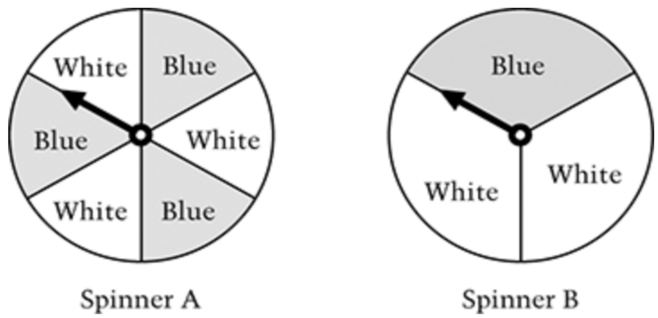
> For $n = k + 1$, $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\left(\frac{1}{2}\right)^{k+1}$ can be expressed as $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\left(\frac{1}{2}\right)^{k}+\left(\frac{1}{2}\right)^{k+1}$. We know from the above statement that $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\left(\frac{1}{2}\right)^{k}$ is equal to $1-\left(\frac{1}{2}\right)^{k}$, so substituting that yields $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\left(\frac{1}{2}\right)^{k+1} = 1-\left(\frac{1}{2}\right)^{k}+\left(\frac{1}{2}\right)^{k+1}$. Simplifying the expression on the right gives us $\frac{2^{k+1}-1}{2^{k+1}}$, or $1-\left(\frac{1}{2}\right)^{k+1}$.

Knowing a variety of approaches to generating a proof and knowing which one to select for a particular circumstance is an important aspect of justifying and proving.

Another element of justifying and proving is evaluating the validity of a purported proof. This involves not only deciding whether a proof is valid in terms of its conclusion, but also deciding whether a given proof relies on correct assumptions, makes use of merited conclusions and logic, and explains the entire statement or conclusion. These skills can be fostered by challenging students to judge the appropriateness of a given argument (e.g., a formal or informal proof;

Knuth, Choppin, & Bieda, 2009). Some NAEP items could be adjusted or expanded to include evaluating the justifications or proofs of others. For instance, the grade 8 NAEP item in Exhibit 3.15 addresses the question of maximizing the probability of landing on blue.

Exhibit 3.15. Grade 8 NAEP Probability Spinners Item



Lori has a choice of two spinners. She wants the one that gives her a greater probability of landing on blue.

Which spinner should she choose?

◯ Spinner A ◯ Spinner B

Explain why the spinner you chose gives Lori the greater probability of landing on blue.

Asking students to explain *why* the spinner they chose gives Lori the greater probability of landing on blue foregrounds justifying. Students could also be given a version of this task in which other students' explanations for choosing Spinner A are provided, and then be asked which of the explanations is the most convincing to them and *why* it convinces them Versions of the examples below might be offered as text, or by avatars, or through video.

1. Andreas says Spinner A has a greater chance for landing on blue because it has three blue sections and Spinner B only has one blue section.
2. Basil says that Spinner A will have a greater probability of landing on blue because the area of two of the blue sections on Spinner A is equal to the area of the one blue section on Spinner B.
3. Calista says that Spinner A has a greater chance of landing on blue because she tried it out. Calista spun each spinner 10 times. For Spinner A, the arrow fell on blue 6 times. For Spinner B, it only fell on blue 2 times.
4. Dora says that Spinner A will have a greater probability because it is one-half blue, but Spinner B is only one-third blue and one-half is more than one-third.

Engaging in justifying and proving is a way for students to explore why a particular assertion must be true. Granted, some proofs might only serve to verify the truth of a statement without helping students understand why; researchers refer to these as "proofs that prove" rather than "proofs that explain" (Hanna, 1990). Certainly not all proofs are explanatory, but in many cases, justifying or evaluating a given argument can help students understand why a conjecture is true. While investigating the reasons a conjecture might be true, students attend to particular features and consider relationships, examine multiple factors that are relevant to the problem statement, return to the meanings of terms and operations, or notice similarity or difference across cases. By exploring these factors, students gain new insight into the conjecture or deepen their understanding of fundamental mathematical ideas.

The grade 8 algebra item in Exhibit 3.16 foregrounds generalizing but could be revised into a justification task. In the item as given, the pattern that the number of diagonals d is equal to the number of sides $n - 3$ is readily apparent from the provided cases. However, adding a prompt asking *why* the equation $d = n - 3$ is a reasonable conjecture for any convex polygon would foreground justifying and proving. A valid justification might involve drawing a few cases, reasoning that from any given vertex one cannot draw a diagonal to itself and one cannot draw a diagonal to the two adjacent vertices (because this makes up two of the sides of the polygon), which means that three of the vertices cannot have diagonals drawn to them while the remaining vertices can.

Exhibit 3.16. Grade 8 NAEP Algebra Generalization Item

From any vertex of a 4-sided polygon, 1 diagonal can be drawn.

From any vertex of a 5-sided polygon, 2 diagonals can be drawn.

From any vertex of a 6-sided polygon, 3 diagonals can be drawn.

From any vertex of a 7-sided polygon, 4 diagonals can be drawn.

How many diagonals can be drawn from any vertex of a 20-sided polygon?

Answer:_________________

The item in Exhibit 3.16 also could be revised into a task to justify why the total number of diagonals that can be drawn for any given convex polygon is $n(n - 3) / 2$. Justifying could take the form of first describing why the number of diagonals that can be drawn from a vertex is $n - 3$ (as above) and then reasoning that since there are n vertices, one could draw $n(n - 3)$ diagonals. However, this would mean that each diagonal would be drawn twice, to and from each vertex. Therefore, in order to avoid double-counting the diagonals, one must divide by 2, yielding the expression $n(n - 3) / 2$. To further illustrate the difference between a proof that proves and one that explains, note that the expression for the total number of diagonals can also be proved by induction. Such a proof by induction would verify the statement without revealing why it is true.

Justifying and proving can help students develop a new and deeper understanding of the mathematics content at hand. Making sense of others' justifications or proofs—and determining their validity—can help students generate new ideas, conjectures, and generalizations, or can support their efforts to develop a new theory to be tested. That is, justifying and proving is an important mode of communication. Proofs can reveal the tools, strategies, modes of thinking, and resources used by those who created them.

NAEP Mathematical Practice 4: Mathematical Modeling

> *Mathematical Modeling: Making sense of a scenario, identifying a problem to be solved, mathematizing it, applying the mathematization to reach a solution, and checking the viability of the solution in developmentally and mathematically appropriate ways.*

Mathematical modeling involves student choice, including the assumptions made in the posing of answerable questions in an open-ended situation. The practice of modeling requires students to make sense of a scenario, identify a problem to be solved, mathematize it, and apply the mathematization to reach a solution and check the viability of the solution. Mathematical modeling also requires discussions and decisions about what is valuable (Burroughs & Carlson, 2019).

At an introductory level, modeling involves steps such as selecting and applying mathematical processes or expressing mathematical concepts and processes (such as mathematical operations) using visual, physical, or symbolic representations. At a more advanced level, a series of processes may be needed to mathematize a messy real-world situation prior to selecting and applying the mathematics. Follow-up work can involve analyzing and evaluating the results obtained from doing the mathematics. A full cycle in the mathematical modeling process includes: (a) identifying the problem; (b) making assumptions that often simplify the problem and then identifying variables; (c) mathematizing the situation; (d) analyzing and assessing solutions; and (e) translating the solution(s) back into the real world and examining their feasibility, and, if not feasible, changing the simplifying assumptions and iterating the process. Finally, if there seems to be a feasible real-world solution, there are two additional steps: (f) implementing the model; and (g) reporting out results (Garfunkel & Montgomery, 2019, pp. 12–13).

It is important to distinguish between the process of mathematical modeling and the noun "model," which is an object and a term sometimes used as a synonym for a mathematical representation. For example, when a line or other function is fitted to a bivariate scatterplot, the function is referred to as a model for the data, meaning a representation of the data. However, the practice of mathematical modeling involves far more than just using a representation. As previously described, mathematical modeling is a multistep process, which may involve aspects of representing, particularly building or interpreting a representation. However, the NAEP Mathematical Practice of Mathematical Modeling is distinct from that of Representing in that the use of representations in modeling is necessarily in service of the overarching purpose of identifying and finding solutions for problems in real-world situations. Items assessing the NAEP Mathematical Practice of Mathematical Modeling focus on multiple steps of the cycle of mathematical modeling driven by that overarching purpose. For example, given an open-ended situation, students could generate questions they would need to explore or identify some assumptions as they begin the modeling process. In such scenarios, students would engage in the first two steps of the modeling process.

Scenario-based tasks are particularly useful in assessing student achievement in the practice of mathematical modeling. Consider the Lunch Problem scenario in Exhibit 3.17 (based on Garfunkel & Montgomery, 2019, pp. 38–42).

Exhibit 3.17. Grade 4 Example: Adaptation of GAIMME Lunch Problem Scenario

> *[Task is introduced through video: A school food service director states during the morning announcements that the school is planning a "Garden Bar" as an option for school lunch <video/image of a garden bar with a variety of fruits and vegetables> The director says, "The cafeteria staff and I would like your input, so we know that the fruits and vegetables included will be eaten. To assist us in our decision-making process, we are establishing a task force to help us gather your suggestions and will take your suggestions into account when making our decision."]*
>
> You volunteer for the task force.
>
> At the first meeting, the team works to determine what they need to know and how to go about gathering that information. Some of the questions your team identifies are:
>
> "How many students are in the school? Do students like some of these choices more than others? Do some of these choices cost more than others? If so, which ones might we have some left over, which might we run out of? Should the school's cost of these items be considered?"

From the scenario launch, several questions might be asked. Students who address these questions would be engaging in components (a) and (b) of the modeling cycle (identifying the problem and making assumptions).

Other tasks built from a similar scenario, about a pizza party for a grade 8 class, could be posed in different ways, depending on the aspect(s) of the modeling process being assessed. For example, grade 8 students could be given the open prompt: "How many and what types of pizzas should be ordered for an 8th grade party?" Some possible questions for students to address as they attempt to model this situation are: "How many students do we expect to feed? How can we find out what types of pizza they like? Should we survey some of the students? How do we decide who to survey? What sizes of pizzas should we order? What is the cost of each size of pizza?" Here students would need to devise a survey (identify the problem) and narrow down to choices of pizza and sizes of pizza (make assumptions; identify variables), and, as they begin to investigate costs of sizes and types of pizza, they would need to create estimates for the cost of the party (mathematize the situation; analyze and assess solutions).

At grade 12, a similar scenario-based open-ended task might include items based on a scenario such as: "What is the best type of computer for the school district to order for students to use in computer labs?" Some possible issues students may need to address as they attempt to model this situation are: "How many computers are needed in a school lab, and how do we know? Is there a break on cost if a large number of computers is purchased at the same time? Which types of classes will need access to the computers? What types of software will be needed for the classes? Do any of the companies offer deals for software along with the computer purchase? How much money can be spent per student?" There are many decisions to be made about what to include and what to assume to address this task. The problem also evokes initial mathematization processes when students ask questions like: “How much money per student?” or “Are there deals for software inclusion or a price break on a large order?”

Exhibit 3.18 is an example where some initial information is provided and students could work to develop a mathematical model (possibly in teams). The first three parts of the task are a scaffold to the modeling-heavy work of parts 4 and 5. Parts 3 and 4 engage students in aspects of modeling components (b), (c), and (d) when identifying variables, mathematizing situations, and analyzing and assessing solutions. Part 5 engages students in components (d), (e), and (g) of the modeling cycle.

Exhibit 3.18. Grade 12 Example: Modeling Income Tax Scenario

A state’s tax model is described below.

- Individuals with an income of $10,000 or less per year pay no income tax.
- Individuals with income greater than $10,000 per year pay a 6% tax on all income over $10,000.

(1) What would a resident who made $40,000 pay in tax? What percent of this resident’s total income is paid in tax?
(2) What would a resident who made $50,000 pay in tax? What percent of this resident’s total income is paid in tax?
(3) Determine a method for calculating the percent of any resident’s total income that is paid in tax.
(4) Is there a highest percent of total income that a resident could pay in tax? Defend your position on this percent.
(5) The state is considering the new tax model described below:
   - Individuals with an income of $10,000 or less per year pay no income tax.
   - Individuals with an income greater than $10,000 per year
     - pay 5% on all income over $10,000 up to $50,000, and
     - pay 7% on all income over $50,000.

   Explain whether or not the new tax model benefits individuals in the state who pay income tax. As part of your response, compare the new tax model to the existing tax model.

Access to digital tools, such as equation editors, graphing tools, and spreadsheet tools, would be important in the assessment of students’ modeling practices on tasks like Exhibit 3.18. For example, in parts 3 and 4, the percent income paid in tax can be expressed as the ratio of tax T to income I, or T/I (identify variables). When students compute the tax on income I, with the given 6% rate after the first $10,000 of income, they arrive at $T = 0.06(I - \$10{,}000)$ (mathematize the situation). A symbolic model for the percent income paid in tax could be $T/I = 0.06(I - 10{,}000)/I$. To answer questions about the highest possible tax rate, students could create a graphical model of the percent income paid in tax as a function of income, I. The mathematization process for this task starts with decisions about using ratios and percent and then could evolve to developing an algebraic expression to model the percent income paid in tax or even a graph of the percent income paid in tax as a function of income (analyzing and assessing the solution).

Modeling processes also often arise in data analysis and statistics. The task in Exhibit 3.19 is an example taken from the online bank of tasks available from Levels of Conceptual Understanding in Statistics (LOCUS, 2019).

Exhibit 3.19. Grade 8 LOCUS Data Modeling Task

The student council members at a large middle school have been asked to recommend an activity to be added to physical education classes next year. They decide to survey 100 students and ask them to choose their favorite among the following activities: kickball, tennis, yoga, or dance. (a) What question should be asked on the survey? Write the question as it would appear on the survey. (b) Describe the process you would use to select a sample of 100 students to answer your question. (c) Create a table or graph summarizing possible responses from the survey. The table or graph should be reasonable for this situation. (d) What activity should the student council recommend be added to physical education classes next year? Justify your choice based on your answer to part (c).

As posed, this task covers the complete modeling cycle from (a) to (g) and closely follows the statistical investigation process as outlined by Bargagliotti and colleagues (2020): identifying a statistical question for investigation, gathering appropriate data, analyzing the data, and communicating the results. The task assesses several content objectives in the data analysis, statistics, and probability area, including posing a statistical question, addressing issues of bias in surveys, and creating tables and graphical representations of data. Though the task as written addresses a full modeling cycle, some parts could be supplied to students and then students could be asked to engage in a narrower aspect of the modeling process.

Although modeling tasks—especially separate aspects of the modeling process—could be posed to individual students, in the workplace mathematical modeling is often done in teams. The importance of preparing students to solve problems is regularly identified as a 21st-century skill. The U.S. Department of Labor, Office of Disability Employment Policy (2010), has noted:

> The ability to work as part of a team is one of the most important skills in today's job market. Employers are looking for workers who can contribute their own ideas, but also want people who can work with others to create and develop projects and plans. (p. 57)

In school mathematics, students already often work together in groups on mathematical tasks, and a mathematical modeling situation provides an inviting context for the use of collaborative tasks. The practice of mathematical modeling is also a natural place to use scenario-based tasks. Many of the sample tasks provided in this section could best be done by groups or pairs of students. When a task is worthy of group effort, the assessment could focus on group responses, solutions, and problem-solving activity. Such an assessment approach is central to the final practice of the NAEP Mathematics Framework, collaborative mathematics.

NAEP Mathematical Practice 5: Collaborative Mathematics

> *Collaborative Mathematics: The social enterprise of doing mathematics with others through discussion and collaborative problem solving whereby ideas are offered, debated, connected, and built-upon toward solution and shared understanding. Collaborative mathematics involves joint thinking among individuals toward the construction of a problem solution in developmentally and mathematically appropriate ways.*

Collaborative mathematics in the world of work refers to the talk and actions people engage in with one another as they participate in a necessary collaboration, where the mathematical task is too complex or messy for an individual to meet its demands alone (Fiore et al., 2017). As a practice, collaborative mathematics exists alongside other mathematical practices. That is, as students work together toward a shared goal, they may also engage in representing, abstracting and generalizing, justifying and proving, and mathematical modeling. Assessing collaborative mathematics requires developing items that foreground and require the doing of mathematics collaboratively, engaging processes that are fundamentally about *joint thinking* (Teasley & Roschelle, 1993). Collectively, these processes include sharing ideas with others; attending to and making sense of the mathematical contributions of others; evaluating the merit of others' ideas through agreement or disagreement; and productively responding to others' ideas through building on or extending ideas and connecting or generalizing across ideas.

Collaborative mathematics processes are largely understood as discursive in nature and occurring through social interaction during mathematical activity. NCTM's policy documents reflect a long-standing focus on discourse and communication. Beginning with the Mathematics as Communication standard (NCTM, 1989) and attention to discourse (NCTM, 1991), mathematics educators have argued that when students write and talk about their thinking, not only do they clarify their own ideas, but they also offer valuable information for assessment.

Given the discursive nature of collaborative mathematics, NAEP Mathematics Assessment items that measure collaborative processes should likewise be discursive in nature, offering students examples of social interaction or imagined utterances around mathematics to which they are tasked to respond in key ways. These include being asked to make sense of others' thinking, express and defend agreement or disagreement, and extend an idea. Tasks might also be genuinely collaborative in nature, asking assessed students to work together in a team during the assessment, such as on a mathematical modeling task.

The discursive nature of collaborative mathematics also means that it is a highly contextualized activity, tied to cultural ways of working together both in and out of the classroom. As stated in the opening of this chapter, while state standards have long included mathematical practices, and collaboration among students has long been emphasized, instruction that engages students in mathematical practices generally, and through collaborative activity in particular, may not yet be pervasive. Without careful attention to opportunities to learn, the assessment may privilege particular out-of-school cultural repertoires for collaboration, particularly around critique.

The assessment of collaborative activity is not new. The Programme for International Student Assessment (PISA), for example, assesses collaborative problem solving, defined as:

> the capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills, and efforts to reach that solution. (OECD, 2017, p. 6)

As illustrated in the components from a PISA scenario-based collaborative problem-solving task (Exhibits 3.20 and 3.21), the task structure involves a dialogue between a team of avatars and the assessed student. The problem task is on the right of the screen, while the running dialogue is on the left (Exhibit 3.20). The assessed student is to choose a discursive response to productively move the collaboration forward. In the example offered in the subsequent screenshots in Exhibit 3.21, one can see that the components of the task emerge as interactional contributions are offered by each avatar (e.g., "Brad") and the assessed student ("you") through item response choices.

Exhibit 3.20. Example PISA Collaborative Problem-Solving Item

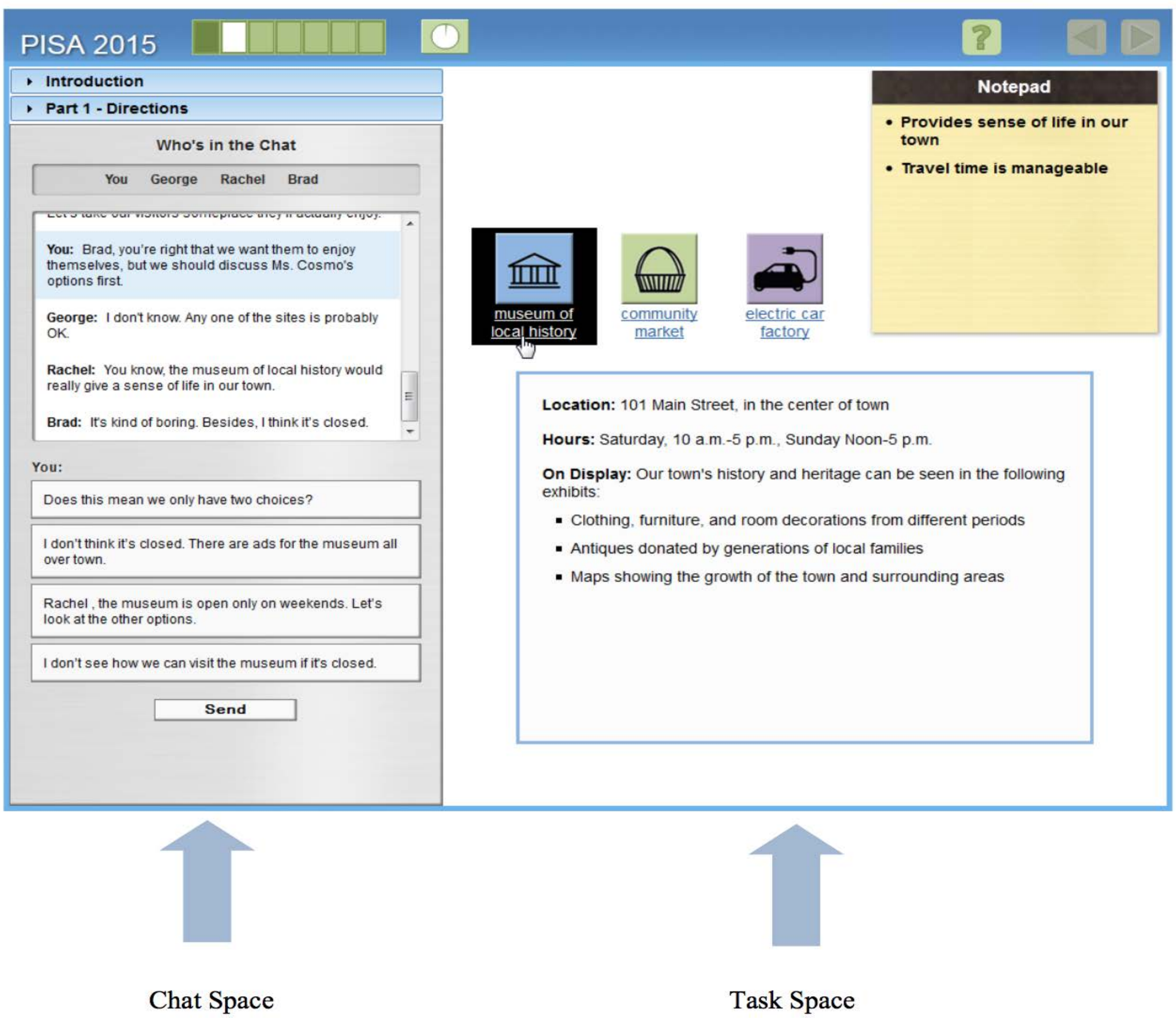
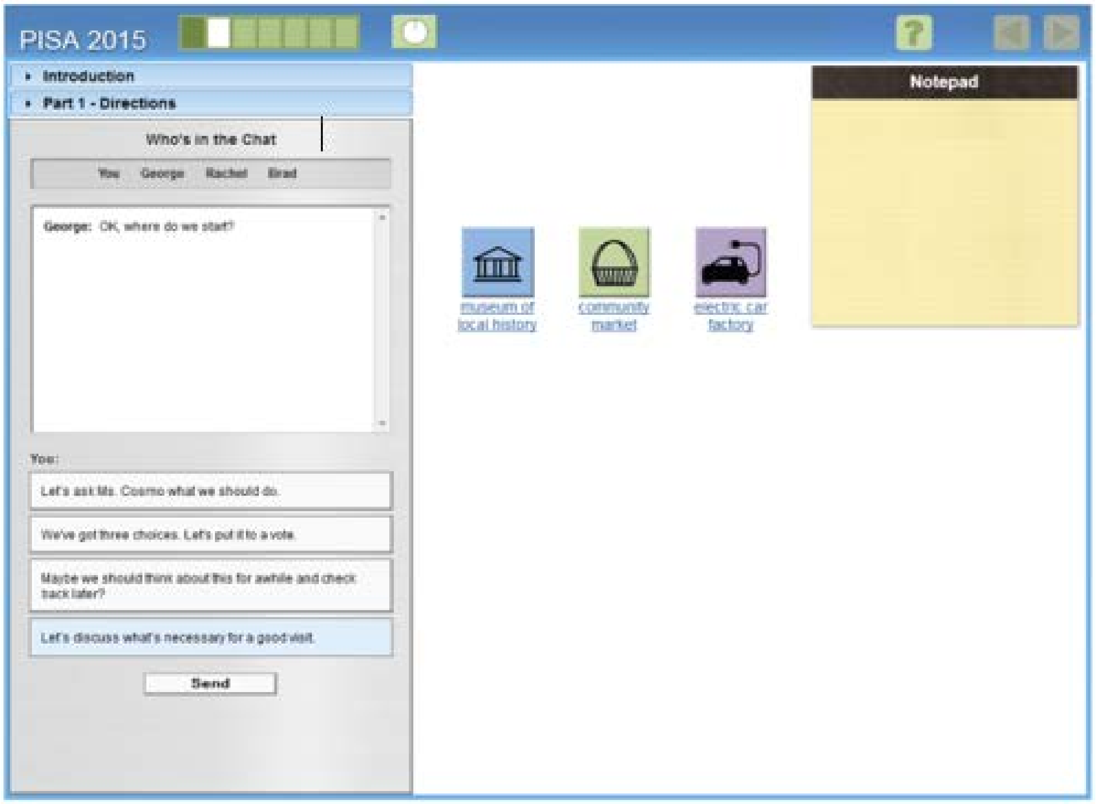
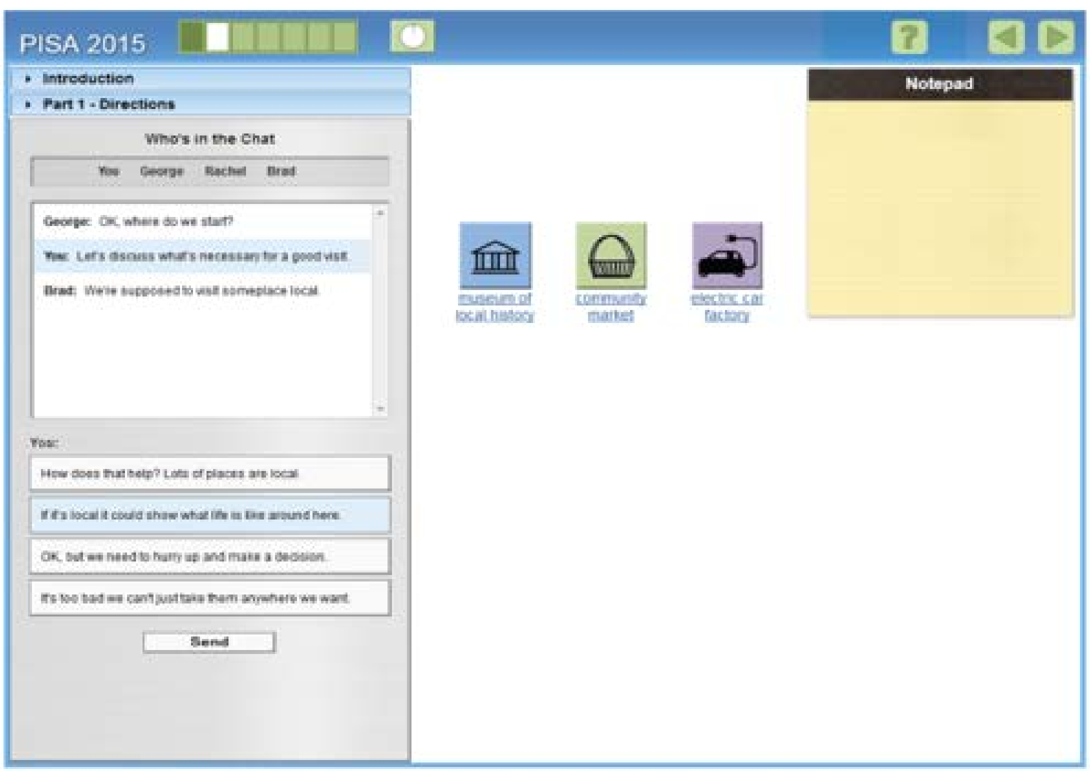


Exhibit 3.21. Example PISA Collaborative Problem-Solving Interaction



Brad mentions that the group is supposed to visit someplace local.



While PISA collaborative problem-solving items are helpful in highlighting discursive assessment, PISA items are not specifically focused on mathematics. Rather, PISA assesses three generic collaborative problem-solving competencies: establishing and maintaining a shared understanding; taking appropriate action to solve the problem; and establishing and maintaining team organization. Additionally, PISA's collaborative problem-solving items are intended to assess problem-solving competencies such as exploring and understanding; representing and formulating; planning and executing; and monitoring and reflecting.

Some of these competencies may apply to collaborative mathematics, but the aim for NAEP is to assess the collaborative processes involved in *mathematics* in particular. The following sections describe three measurable skills involved in collaborative mathematics:

- attending to and making sense of the mathematical contributions of others,
- evaluating the mathematical merit of the contributions of others, and
- responding productively to others' mathematical ideas.

Attending to and Making Sense of the Mathematical Contributions of Others

Collaborative mathematics begins with the sharing of ideas in the form of a conjecture or other contribution that is meant to be communicated to others. A first joint act is made up of both this sharing and how others attend to the conjecture and make sense of it (Forman, Larreamendy-Joerns, Stein, & Brown, 1998). To do so, students must establish a shared understanding about what the problem is and how the problem is being interpreted (Lerman, 1996).

While classroom studies document the importance of making sense of peers' ideas during collaborative mathematics activity, most research on the discursive processes in making sense of student thinking has looked at teacher talk moves rather than student talk moves (Chapin, O'Connor, O'Connor, & Anderson, 2009). These moves are nevertheless relevant in framing how students make sense of one another's mathematical thinking. For example, people *elicit* and *probe* ideas. Individuals then express and check personal understanding of another's thinking by repeating or *revoicing* the idea (Enyedy et al., 2008). During a collaborative mathematics assessment task, students can elicit, probe, and revoice peers' ideas to demonstrate and check for understanding.

Revoicing is a particularly powerful discursive opportunity to assess whether a student has understood the mathematical contribution of others. Revoicing is defined as "when one person re-utters another's contribution through the use of repetition, expansion, or rephrasing" (Enyedy et al., 2008, p. 135). From an assessment perspective, students can be asked to revoice (or put into their own words) the expressed mathematical ideas of another student/an avatar, or to justify its mathematical appropriateness.

Evaluating the Mathematical Merit of the Contributions of Others

Once students attend to and make sense of the thinking of others, they must evaluate the mathematical reasonableness of their peers' mathematical contributions. Generally, students express their evaluation of the mathematical reasonableness of an idea through agreement or disagreement, including some explanation or justification. Agreeing or disagreeing emerges out of shared understanding (Nathan, Eilam, & Kim, 2007). This skill is critical to the development of productive mathematical argumentation. Experimental and classroom studies have found that students' ideas can be evaluated and become influential due to issues of status or authority rather than mathematics sense-making (Cohen & Lotan, 1997; Engle, Langer-Osuna, & McKinney de Royston, 2014).

Exhibit 3.22 shows a grade 4 SBAC (2018) item suited to assess the collaborative skill of evaluating the mathematical merit of the contributions of others. In the item, the assessed student is offered a strategy for solving a problem by an imagined student, Connor. The assessed student is asked to evaluate Connor's stated strategy and decide whether or not he is correct and why. Digitally based administration of this and similar items could provide the assessed student the opportunity to read or hear (through voiceover) Connor's own utterances, make sense of Connor's thinking, and then choose an evaluation with explanation

Exhibit 3.22. Adapted Grade 4 SBAC Number Properties Collaborative Mathematics Item

Together, you and Connor are finding 8 × 16.

Connor says, "We can find the product if we multiply 8 and 15 and then add 8."

Which sentence could you say to Connor to **best** explain that his statement is correct or incorrect?

A. I think you are incorrect, because we should add 16 instead of 8.
B. I think you are correct, because 15 is an easier number to multiply by than 16.
C. I think you are correct, because 8 × 16 is the same as 15 groups of 8, plus 1 group of 8.
D. I think you are incorrect, because 8 × 16 is the same as 4 groups of 8, plus 4 groups of 8.

Exhibit 3.23 shows another grade 4 item from the SBAC collection. Like the previous example, the item begins with a collaborative situation within which the assessed student is offered a glimpse into the thinking of an imagined peer, Jose. Here, Jose offers a conjecture about number. The assessed student is asked to critique Jose's conjecture by offering a counterexample that proves Jose's statement false. A digitally based assessment means the assessed student could have the opportunity to read or hear (through voiceover) Jose's own utterance, make sense of Jose's thinking, and then complete a sentence that shows why Jose's statement is false. Although the item tells the student that Jose's statement is incorrect, the assessed student needs to understand Jose's statement before responding. The item also addresses the practice of justifying and proving, through the required completion of a counterexample to refute Jose's statement.

Exhibit 3.23. Adapted Grade 4 SBAC Number Properties Collaborative Mathematics Item

> You and Jose talk about the number of factors all whole numbers have.
>
> Jose says that all whole numbers except 1 have an even number of factors because factors always come in pairs.
>
> Jose's statement is incorrect.
>
> Complete the sentences to help Jose see that his statement is not always correct. Drag numbers into the empty boxes to complete the sentences.
>
> 1
> 2
> 3
> 4
> 5
> 6
> 7
> 8
> 9
> 10
>
> What about the number ☐?
>
> It has ☐ factors.

Consider, again, Exhibit 3.13 (p. 63), a grade 12 NAEP Mathematics Assessment item also suited to assess collaborative mathematics. In the item, the assessed student is given an exchange by two imagined students, Tracy and Pat. That is, the assessment happens in the context of examining the justifying activity of Pat. Tracy offers a conjecture about which Pat expresses and explains disagreement. Assessed students are asked to evaluate these utterances and decide which is correct and to explain their evaluation. Again, an assessed student has the opportunity to read or hear (through voiceover) Tracy and Pat's own utterances. This conversational format is preferable to an offer of paraphrased positions that the assessed student is tasked to evaluate.

Responding Productively to Others' Mathematical Ideas

A third mathematics-specific collective process involves responding productively to others' mathematical ideas. In particular, students learn to build on, extend, and connect across mathematical ideas. These discursive acts depend and build on the acts of making sense of and evaluating others' mathematical thinking. Once a shared mathematical idea is understood, students can further contribute to the mathematical discussion by acting upon those shared ideas. Connecting across students' mathematical ideas is a core discursive component of productive collaborative mathematics (Stein, Engle, Smith, & Hughes, 2008). By connecting ideas, students are able to notice and explain how two seemingly different strategies hold the same mathematical ideas. Students also build on or extend an idea through new examples, next steps, or logical deductions.

Balance of Mathematical Practices

The target percentage ranges of items for each NAEP Mathematical Practice are given in Exhibit 3.24. Most NAEP Mathematics Assessment items will feature one of the five NAEP Mathematical Practices (55 to 85 percent). The range of 55 to 85 percent allows flexibility in assessment and item development across grades 4, 8, and 12, while also ensuring that the majority of the assessment is designed to capture information on students' knowledge while they engage in NAEP Mathematical Practices. All NAEP Mathematical Practices will be represented in all grades and at least at the minimal levels. The relative emphasis on justifying and proving is based on its centrality across a range of mathematical activity; for example, the SBAC assessment targets justifying across multiple content categories, including modeling and data analysis, and communicating reasoning at every grade level.

Exhibit 3.24. Percentage Distribution of Items by NAEP Mathematical Practice

NAEP Mathematical Practice Area	Percentage of Items
Representing	10–15
Abstracting and Generalizing	10–15
Justifying and Proving	15–25
Mathematical Modeling	10–15
Collaborative Mathematics	10–15
Other	15–45

The remaining balance of items (15 to 45 percent) fall into the "Other" category and will assess knowledge of content without the item being designed to also assess a particular NAEP Mathematical Practice. Examples might include items that emphasize mathematical facts or procedural fluency or items that target practices that are not included in the five identified for the NAEP Mathematics Assessment. As noted earlier in this chapter, this could also include items that focus on algorithms, precision, or tool use.

Challenges

Together, the past several decades of research on mathematics thinking and learning and the consensus judgment of experts in mathematics education provide strong warrants for incorporating mathematical practices into the NAEP Mathematics Assessment. Despite widespread consensus on their importance, there are many challenges to assessing the NAEP Mathematical Practices. One is the interrelated nature of mathematical practices. Second, there is not consensus on how to define, let alone assess, mathematical practices. Finally, given the state of research and item development, it will be challenging to have sufficient numbers of items that assess student achievement with each NAEP Mathematical Practice, presenting challenges to reporting results on the Practices.

Although these challenges are formidable, they are not insurmountable. Existing state assessment programs include mathematical practices in their assessments. PISA has also been assessing mathematical practices for some time. Challenges can be addressed as the mathematical practices are incorporated into the 2026 NAEP Mathematics Assessment and refined over successive administrations. In addition, a special study to examine ways to report on mathematical practices to the general public is described in the *Assessment and Item Specifications* document. Despite these challenges, NAEP is clearly advancing mathematical practices as a core component of student achievement in mathematics, with the opportunity to become a leader in designing valid ways to assess the practices and report the results.

Exhibit 3.25A. Practices and Content Illustrations—Grade 4

In each cell, practice descriptors are included for a particular content area. The entries in this table are intended to be illustrative, not comprehensive.

Representing Grade 4				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Represent numbers or operations using visual models (e.g., base 10, number lines, fraction strips). Recognize, translate between, interpret, and compare written, numerical, and visual representations of large numbers (e.g., thousands).	Select appropriate units related to representing or measuring an attribute of an object. Create visual representation of measurements or relationships between measurements.	Draw or sketch figures from a written description. Represent or describe figures from different views. Use a geometric model of a situation to draw conclusions.	Create a visual graphical, or tabular representation of a given data set. Compare and contrast different visual and graphical representations of a univariate distribution.	Recognize, describe, or extend numerical and geometric patterns using tables, graphs, words, or symbols. Translate between different representations of numerical expressions using symbols, tables, diagrams, or written descriptions.

Exhibit 3.25A. Practices and Content Illustrations—Grade 4 (continued)

Abstracting and Generalizing Grade 4				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Identify patterns in numbers or figures and generalize patterns in written or pictorial forms. Describe or extend a pattern or relationship to a larger set of numbers. Find structural relationships among sets of numbers. Generalize understanding of place value.	Make generalizations about areas of squares or rectangles. Extend quantified attributes to a larger set.	Generalize geometric properties by making connections across different figures and families of figures (e.g., triangles, quadrilaterals, polygons, polyhedra). Extend a geometric relationship from one or more figures to a family of figures.	Interpret graphical or tabular representations of data in terms of generalized phenomena (e.g., middle or median, range, mode, or shape). Make general conclusions about graphs of single sets of data (e.g., pictographs, bar graphs, dot plots).	Generalize a pattern appearing in a sequence or table, using words or symbols. Given a description, extend a pattern or sequence.

Justifying and Proving Grade 4				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Defend or counter claims about why a numerical relationship or pattern is valid or will always hold. Evaluate the appropriateness of an argument provided about properties or operations.	Defend or counter a claim about physical attributes, comparisons, or measurement properties. Choose a counterexample that disproves a claim about properties such as area, length, or volume.	Validate geometric conjectures (e.g., distinguish which objects in a collection satisfy a given geometric property and defend choices).	Evaluate the characteristics of a good survey and justify a survey's validity. Defend or counter conjectures offered based on a data set.	Make and justify conclusions and generalizations about numerical relationships. Given a pattern or sequence, construct, explain, or justify a rule to generate the terms of the pattern or sequence.

Exhibit 3.25A. Practices and Content Illustrations—Grade 4 (continued)

Mathematical Modeling Grade 4				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Use physical or virtual materials to build a model of a number pattern or to predict or estimate results of a continued pattern. Select and defend an appropriate method of estimation as a model for an estimation problem. Select appropriate properties or operations that can be used to build a model of a situation or solve a problem.	Identify the attribute(s) appropriate to measure in a given situation. Mathematize a contextual measurement situation to lead to a solution.	Use existing geometric models to solve mathematical or real-world problems.	Identify a statistical question to investigate in a given, open-ended or data-rich situation.	Identify a mathematical problem from a given situation that could be modeled numerically. Identify the variables needed to create an algebraic model of a situation.

Exhibit 3.25A. Practices and Content Illustrations—Grade 4 (continued)

Collaborative Mathematics Grade 4				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Add to a numerical model provided by others to complete a mathematical task. Evaluate others' interpretations of numbers from real-life contexts. Analyze the effect of another's estimation method on the accuracy of results.	Evaluate the validity of a measurement claim posed by others. Analyze others' solutions and suggest a critique of their solutions in a situation involving measurement. Attend to and make sense of the mathematical contributions of others in a situation involving measurement (e.g., revoice the work of others to clarify meaning of choice of measurement units).	Express and justify agreement or disagreement with a claim made by others in a geometric problem situation. Build on the work of others to geometrically model a situation.	Recognize and critique misleading arguments from data (e.g., from media or other people).	Verify the conclusions of others using algebraic/numerical properties.

Exhibit 3.25B. Practices and Content Illustrations—Grade 8

In each cell, practice descriptors are included for a particular content area. The entries in this table are intended to be illustrative, not comprehensive.

Representing Grade 8				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Represent word problems through visual models. Recognize, apply, create, or translate across multiple representations of fractions (e.g., visual models of equivalent fractions) and rational numbers (decimals, fractions, percents).	Select or use appropriate measurement instruments to determine the attributes of an object. Create visual representation of measurements or relationships between measurements.	Represent or describe figures from different views. Visualize and solve problems using geometry (e.g., using 2-D representations of 3-D objects). Use a geometric model of a situation to draw conclusions. Represent problem situations with geometric models to solve mathematical or real-world problems.	For a given set of data, create a visual, graphical, or tabular representation. Compare and contrast different visual and graphical representations of univariate and bivariate data. Justify the use of a particular representation of data over another. Interpret visual representations to compare data sets, to draw inferences, or to make conclusions across two or more distinct data sets. Create and use scatterplots to represent the relationship between two variables and to estimate the strength of the relationship (strong, weak, none).	Use or create a graphical representation of a situation to draw conclusions. Translate between different representations of expressions using symbols, graphs, tables, diagrams, or written descriptions.

Exhibit 3.25B. Practices and Content Illustrations—Grade 8 (continued)

Abstracting and Generalizing Grade 8				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Determine an expression for a recursive pattern. Generalize, describe, or compare numerical properties and operations across different domains. Extend a pattern or relationship to a larger set of numbers. Find and generate structural relationships among sets of numbers. Generalize findings about rational and irrational numbers.	Extend quantified attributes to a larger set. Make connections between representations of different measurement systems.	Describe the general effects of dilations, translations, and rotations for two-dimensional figures. Identify common elements across different figures and families of figures (e.g., triangles, quadrilaterals, polygons, polyhedra). Extend a geometric relationship from one or more figures to a family of figures.	Interpret graphical or tabular representations of data in terms of generalized phenomena (e.g., shape, center, spread, clusters). Generalize trends in data to suggest interpretations or infer conclusions.	Generalize a pattern appearing in a sequence, table, or graph using words or symbols. Develop general rules for translating functions and graphs. Create connections across representations.

Exhibit 3.25B. Practices and Content Illustrations—Grade 8 (continued)

Justifying and Proving Grade 8				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Defend a claim about why a numerical relationship or pattern is valid or will always hold. Find a counterexample to refute a claim about number properties or operations. Evaluate the appropriateness of a provided argument about properties or operations.	Defend a claim about physical attributes, comparisons, or measurement properties. Evaluate the validity of a provided argument making use of measurement. Find a counterexample to disprove a claim about properties such as area, length, or volume.	Verify properties of rotations, reflections, or translations. Create, test, and validate geometric conjectures (e.g., distinguish which objects in a collection satisfy a given geometric definition and defend choices). Defend claims about similarity of two-dimensional figures. Analyze a provided argument about geometric attributes or relationships.	Evaluate the characteristics of a good survey or of a well-designed experiment and defend the validity of surveys or experiments. Offer counter arguments in relation to conjectures about bivariate data.	Develop a valid mathematical argument based on properties of slope and intercept for linear functions. Justify functional relationships across different representational forms, such as tables, equations, verbal descriptions, or graphs.

Exhibit 3.25B. Practices and Content Illustrations—Grade 8 (continued)

Mathematical Modeling Grade 8				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Build a model of a situation for an estimation problem. Communicate and defend a decision about a physical or virtual model involving number and/or operation to an audience for feedback.	Mathematize a contextual measurement situation to lead to a solution. Evaluate the reasonableness of a model unit for an attribute in a real context.	Visually model the effects of successive (or composite) transformations of figures in the plane. Construct geometric models using physical or virtual materials to solve mathematical or real-world problems.	Identify a statistical question to investigate in a given, open-ended or data-rich situation. Create or use a statistical model to answer a statistical question or make a prediction about a data set. Create or use a statistical model to assess the validity of a statistical claim.	Identify the variables needed to create an algebraic model of a situation. Write algebraic relationships, expressions, equations, or inequalities to model real-world situations.

Exhibit 3.25B. Practices and Content Illustrations—Grade 8 (continued)

Collaborative Mathematics Grade 8				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Build on a numerical model provided by others to complete a mathematical task. Analyze the effect of another's estimation method on the accuracy of results. Reflect on the work of others to extend a numerical pattern.	Evaluate the validity of a measurement claim posed by others. Engage in joint thinking to reach consensus about a measurement situation. Analyze others' solutions and suggest a critique of their solutions in a situation involving measurement.	Express and justify agreement or disagreement with a claim made by others in a geometric problem situation. Build on the work of others to geometrically model a situation. Evaluate the merit of others' geometric ideas. Connect across geometric ideas contributed by others in a problem-solving situation.	Choose a worthwhile statistical question from a set offered by others about a problem situation or context involving data. Recognize and critique misleading arguments from data (e.g., from media or other people). Revoice the work of others in addressing a statistical or probabilistic situation. Analyze the models constructed by others to evaluate a new data set.	Verify the conclusions of others using algebraic properties.

Exhibit 3.25C. Practices and Content Illustrations—Grade 12

In each cell, practice descriptors are included for a particular content area. The entries in this table are intended to be illustrative, not comprehensive.

Representing Grade 12				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Create and justify solutions to word problems through numeric representations and operations. Represent, interpret, or compare expressions or problem situations involving absolute values.		Represent or describe figures from different views. Visualize and solve problems using geometry (e.g., using 2-D representations of 3-D objects). Represent problem situations with geometric models to solve mathematical or real-world problems.	For a given set of data, create a visual, graphical, or tabular representation of the data. Compare and contrast different visual and graphical representations of univariate and bivariate data. Interpret visual representations to compare data sets, to draw inferences, or to make conclusions across two or more distinct data sets. Create and use scatterplots to represent the relationship between two variables and to estimate the strength of the relationship (strong, weak, none).	Use or create a graphical representation of a situation to draw conclusions. Translate between different representations of expressions using symbols, graphs, tables, diagrams, or written descriptions. Express linear and exponential sequences in recursive or explicit forms given a table.

Exhibit 3.25C. Practices and Content Illustrations—Grade 12 (continued)

Abstracting and Generalizing Grade 12				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Determine a generalized expression for a recursive pattern. Extend properties of numbers from one system to another (for instance, extend the properties of exponents to rational exponents). Generalize, describe, or compare numerical properties and operations across different domains or number systems. Extend a pattern or relationship to a larger set of numbers. Find and generate structural relationships among sets of numbers.	Generalize the effect of proportions and scaling for area and volume. Extend trigonometric formulas to determine triangle unknowns.	Generalize relationships such as congruence, similarity, or orientation between figures and their images under transformation. Extend a geometric relationship from one or more figures to a family of figures. Develop generalizations about transformations that preserve the area or volume of figures.	Interpret graphical or tabular representations of data in terms of generalized phenomena (e.g., shape, center, spread, clusters). Organize and display data in order to recognize and make inferences from patterns in the data. Notice patterns of outcomes in a probability situation. Generalize trends in data to suggest interpretations or infer conclusions. Develop generalizations about how linear transformations of one-variable data affect mean, median, mode, range, interquartile range, and standard deviation.	Extend and generalize numerical patterns, including arithmetic and geometric progressions. Compare and generalize properties of linear, quadratic, rational, and exponential functions. Identify commonalities within and across function families. Develop general rules for translating functions and graphs. Create connections across representations.

Exhibit 3.25C. Practices and Content Illustrations—Grade 12 (continued)

Justifying and Proving Grade 12				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Find a counterexample to refute a claim about number properties or operations. Prove numerical relationships through developing deductive arguments, engaging in proof by exhaustion, or employing mathematical induction. Evaluate the validity of a provided argument about properties or operations. Analyze or interpret a proof by mathematical induction about the properties of numbers. Justify relationships between properties of number systems, including natural numbers, integers, rational numbers, real numbers, and complex numbers.	Justify or prove a claim about physical attributes, comparisons, or measurement properties. Explain why a given attribute can be appropriately measured by the chosen quantity and unit. Evaluate the validity of a provided argument making use of measurement. Find a counterexample to disprove a claim about properties such as area, length, or volume. Prove conjectures about trigonometric identities.	Justify relationships of congruence and similarity; apply these relationships using scaling and proportional reasoning. Create, test, and validate geometric conjectures (e.g., distinguish which objects in a collection satisfy a given definition and defend choices). Analyze a provided argument about geometric attributes or relationships. Use given definitions and theorems to prove geometric conjectures. Develop justifications and proofs that rely on a variety of representational modes (e.g., two-column, paragraph). Discuss the implications that a definition of a type of figure has on the figure properties.	Critique the validity of surveys or experiments. Justify or prove conjectures about probability. Create and explore counting arguments in order to develop and justify conjectures.	Create, validate, and justify conclusions and generalizations about functional relationships. Verify a conclusion using algebraic properties. Prove algebraic relationships through developing deductive arguments, finding counterexamples, engaging in proof by exhaustion, and employing mathematical induction.

Exhibit 3.25C. Practices and Content Illustrations—Grade 12 (continued)

Mathematical Modeling Grade 12				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Select appropriate properties or operations that can be used to build a model of a situation or solve a problem. Create a physical or virtual model involving number and/or operation.	Select or use a model unit for an attribute to be measured and defend the use of that unit. Mathematize a contextual measurement situation to lead to a solution. Create a model to convert between two measurement systems. Construct scale drawings to be used as measurement models of objects in problem situations.	Create a geometric model of a physical object. Discuss differences in solutions caused by having used a simplified model. Use existing geometric models to solve mathematical or real-world problems. Visually model the effects of successive (or composite) transformations of figures in the plane. Construct geometric models using physical or virtual materials to solve mathematical or real-world problems. Predict the results of combining, subdividing, and transforming geometric figures.	Identify a statistical question to investigate in a given, open-ended or data-rich situation. Use a statistical model to answer a statistical question or make a prediction about a data set. Create a probability model to calculate or estimate the probability of an event. Compare and contrast theoretical probabilities with results from experimental probabilities in a simulation.	Identify a mathematical problem from a given situation that could be modeled algebraically. Identify the variables needed to create an algebraic model of a situation. Write algebraic relationships, expressions, equations, or inequalities to model real-world situations. Revise an existing algebraic model based on introducing new variables or parameters. Build or apply a mathematical model of a financial situation (e.g., a monthly family budget, or a car loan).

Exhibit 3.25C. Practices and Content Illustrations—Grade 12 (continued)

Collaborative Mathematics Grade 12				
Number Properties and Operations	**Measurement**	**Geometry**	**Data Analysis, Statistics, and Probability**	**Algebra**
Build on a numerical model provided by others to complete a mathematical task. Analyze the effect of another's estimation method on the accuracy of results. Reflect on the work of others to extend a numerical pattern. Evaluate the mathematical reasonableness of a peer's mathematical contribution.	Evaluate the validity of a measurement claim posed by others.	Express and justify agreement or disagreement with a claim made by others in a geometric problem situation. Attend to the contributions of others in collaboratively generating a geometric proof. Build on the work of others to geometrically model a situation. Generalize across geometric ideas contributed by others in a problem-solving situation.	Revoice/restate the work of others in addressing a statistical or probabilistic situation. Analyze the models constructed by others to evaluate a new data set.	Verify the conclusions of others using algebraic properties.

CHAPTER 4

OVERVIEW OF THE ASSESSMENT DESIGN

This chapter provides an overview of the major components of the mathematics assessment design, which includes the types of assessment tasks and item formats and how they can be used to expand the ways in which students are asked to demonstrate what they know and can do in mathematics. In addition, this chapter describes how the assessment is distributed across the five mathematics content areas described in Chapter 2 and the five NAEP Mathematical Practices in Chapter 3. The 2026 Framework intentionally emphasizes increased access for students—including English language learners and students with disabilities—to demonstrate their mathematics understanding. Scholarship has demonstrated that students of various ethnic, racial, economic, and cultural backgrounds have salient differences that matter to the format and design of assessment items for inclusiveness (Solano-Flores, 2011). In particular, the 2026 NAEP Mathematics Assessment will continue to use concepts of universal design for assessment to increase inclusiveness and assessment validity (Thompson, Johnstone, & Thurlow, 2002).

Previous NAEP Mathematics Assessments included only discrete items, which stand alone or comprise a composite item. Discrete items consist of selected response and constructed response item types. In order for students to demonstrate what they know and can do with respect to the range of mathematics content knowledge and NAEP Mathematical Practices in this framework, the 2026 NAEP Mathematics Assessment includes a new item assessment format: scenario-based tasks. Scenario-based tasks have both context and extended storylines to provide opportunities to demonstrate facility with the integrated nature of mathematics content knowledge and NAEP Mathematical Practices.

Two fundamental aims motivate the expansion. First, there is a need to ground the NAEP assessment in relevant tasks and familiar contexts to provide a better measure of student content knowledge and mathematical practices (Eklöf, 2010). Second, by expanding item types and thoughtfully using technology, the NAEP Mathematics Assessment continues to provide greater access to all students, diversifies the ways in which student achievement can be recognized and measured, and more robustly assesses both what students know and what they can do. For example, graphics can be presented in color with greater clarity and with a tool to zoom in and out (Sireci & Zenisky, 2006).

Technology provides opportunities for assessment, but with each opportunity come myriad constraints and repercussions that must be considered. For example, introducing a new format for items on the NAEP Mathematics Assessment that is interactive or discussion-based requires that great care be taken to ensure that the design is accessible to students, that students have ample time to understand how to engage with the item, and that students have had opportunities to experience the task type. Familiarity with digital technology in general, and with specific digital tools in particular, can influence student performance (Dunham & Hennessy, 2008). Other potential threats to assessment validity are the accessibility of tools and the affordances for students with and without certain disabilities. Due to differential access to, use of, and outcomes stemming from student experiences with technologies in and out of school (Warschauer & Matuchniak, 2010), development work should address known and potential implementation challenges and identify ways to mitigate issues of access in doing the assessment that could occur in under-resourced communities (Warschauer, 2016). A goal of the NAEP Mathematics Assessment is not to disadvantage students by virtue of the assessment's technology.

Types of Tasks, Items, and Supporting Tools

The 2026 NAEP Mathematics Assessment will include existing and new discrete items as well as scenario-based tasks.

Scenario-Based Tasks

The goal of scenario-based tasks is to provide evidence of students' ways of knowing and doing mathematics. Current and future NAEP Mathematics Assessments can take advantage of evolving digital technologies to create the next generation of scenario-based tasks, as well as yet-to-be-imagined items and tasks. Other NAEP frameworks have set a foundation for scenario-based tasks. For example, since 2009 the NAEP Science Framework has called for the use of interactive computer tasks, and the NAEP Technology and Engineering Literacy (TEL) Framework has done so since its start in 2014 (Governing Board, 2014b, 2014c). Examples of scenario-based TEL tasks can be found online (Governing Board, 2014d).

The defining features of the scenario-based task for the 2026 NAEP Mathematics Assessment are an authentic context, in which students can imagine themselves, with a motivating question or goal, along with item design that supports exploration. The motivating goal for a scenario-based task might be to solve a particular problem or to complete a certain mission within the scenario. The goal provides the driving rationale for the tasks that the student will perform. It offers a storyline that helps build needed background, defines the task's relevance and coherence, and motivates the student to engage with the scenario-based task.

Within one scenario-based task, a student may complete multiple items that vary in format, with both constructed and selected response item types (details of item types are provided in the next section). Within a scenario-based task, each item is in some way related to, or builds on, the next item as part of the cohesive experience. Such tasks may be well suited to addressing the intersecting nature of the mathematics content and the NAEP Mathematical Practices illustrated in Exhibits 3.25A–3.25C at the end of Chapter 3. Scenario-based tasks may also be especially well suited to measuring the highly iterative or interactional nature of the NAEP Mathematical Practices described in Chapter 3.

An advantage of digital delivery of the assessment is that scenario-based tasks can use multimedia (e.g., images, video, and animation, in addition to future technologies) to present the settings for the assessment items. As a result, non-mathematical linguistic demand might be reduced while mathematical rigor is maintained. Multimedia can also better scaffold the background understanding that examinees may need to complete a given item. For example, video segments or animations that a student observes, along with text, numbers, and graphics, can convey information necessary for the task to be accomplished. In developing such scenario-based tasks, related design decisions should serve a particular purpose and not be extraneous or presented simply for visual interest. While in many cases relevant multimedia content can have a positive impact on student engagement and performance, it is also possible that it may introduce competition of attention between visual and auditory channels (Fawcett, Risko, & Kingstone, 2015). When multimedia content is included in a scenario-based task, developers need to ensure that the multimedia content is used productively and minimizes such competition.

Within a scenario-based task, students are given opportunities to select tools from a toolkit and use them to solve problems. For example, students might be asked to select a graphing or spreadsheet tool or to use a simulation. Various digital and physical tools may be made available, depending on the scenario. These might take the form of chat/texting, or presentation tools for communication tasks, if deemed relevant to the mathematical understanding being assessed.

When designing tools for a scenario-based task, it is necessary to determine which elements of a tool are needed for the activities in the scenario and which features are used by students. For example, only those functions of a spreadsheet tool that are directly relevant to a given item might be provided. It is not necessary to provide all of the other features of the spreadsheet tool. In fact, including every feature could be distracting to students and could produce measurement error. Additionally, students are not expected to know how to use all tools in a scenario-based task prior to starting the task. In these cases, instructions and practice using the tool are embedded in the task before the tool is needed or used to complete the task.

An important consideration for assessment developers when designing scenario-based tasks is to ask what is gained through the selection of a scenario as assessment context. A robust scenario will allow examinees to interact with task components in multiple ways, explore alternative outcomes and explanations, find multiple solution paths, and demonstrate their thinking. Students could also evaluate the outcomes of the choices they make and convey their understanding of mathematical concepts in diverse ways. For example, one scenario-based task may engage students in a range of mathematical practices and foreground one content area.

Interactive scenario-based tasks can elicit rich data, providing evidence of NAEP Mathematical Practices that are difficult to measure with more conventional items and tasks. For example, measuring collaboration has long been a challenge in assessment. Novel methodological approaches have explored discipline-specific student collaborative activity through the use of performance outcomes and process data from scenario- and simulation-based collaborative assessment (Andrews et al., 2017). These approaches can be used to better assess the NAEP Mathematical Practice of Collaborative Mathematics.

As illustrated in the PISA example in Chapter 3 (see Exhibits 3.20–3.21), validated scenario-based tasks that assess collaborative problem solving already exist. In that example, the task was structured as a dialogue with a collaborative team made up of avatars and assessed students in a way that is nearly impossible to do using only discrete item sets. In contrast, Exhibit 4.1 (based on a grade 8 *Stacking Chairs* task from the Silicon Valley Mathematics Initiative [2016]) illustrates a set of discrete items that are scenario-based, presented in a non-digital environment. Notably lacking from this example are supporting multimedia and tools.

Exhibit 4.1. Grade 8 Scenario Example

You, Lee, and Pat are the team organizing the spring concert at your school. The school has a large room with a stage but the team will need to arrange for renting chairs from a local company. The chairs must be put in a storage room before the concert. The chairs can be stacked. The team stacked some chairs and measured the heights of the stacks. Below are the notes the team made.

The height of stacked chairs

5 chairs are 51 inches high

3 chairs are 45 inches high

8 chairs are 60 inches high

1. How tall are two chairs stacked together? _______ inches

Lee suggests the chairs be stacked in groups of 10.

2. How tall is a stack of 10 chairs? _______ inches
   Show how you figured it out.

The team decides that groups of 10 chairs will take up too much floor space. The team wants an equation to know how tall a stack will be if you know the number of chairs.

3. Write an equation to find the height, *y*, if the number of chairs in a stack is *x*.

4. Explain how Pat can use the equation you wrote to determine the height of 28 chairs.

The storage room is 15 feet tall. Three feet of space above the stack of chairs is needed (to take chairs off the stack).

5. How many chairs can be in a stack and still fit in the storage room? _____ chairs
   Show how you figured it out.

6. There will be 200 chairs for the audience. What else would the team need to know in order to determine whether or not all 200 chairs will fit in the storage room? Why is the information needed?

Due to their capacity to replicate authentic situations (i.e., experiences that students may encounter in their lives), scenario-based tasks have the potential to provide a level of accessibility and support for student engagement with the assessment that other types of assessment tasks do not. Additionally, scenario-based tasks provide opportunities to simultaneously assess multiple practices or content areas. However, a block of scenario-based tasks may provide less measurement information than a block of discrete items in the same amount of assessment time; scenario-based tasks typically require a longer duration to reach optimal reliability (Jodoin, 2003).

Scenario-based tasks will take students about 10–20 minutes to complete. Longer scenario-based tasks may include a greater number of embedded assessment requirements and items to which a student is asked to respond. The discussion of the balance of item types later in this chapter provides a general range to allow item developers greater flexibility to fulfill assessment design blocks.

Item Types

Since 1992, the NAEP Mathematics Assessment has used two types of items: multiple choice and constructed response. In 2017, the term "multiple choice" was revised to "selected response" to account for the wider range of item formats available (e.g., matching) with digitally based assessments. Selected response items require a student to select one or more response options from a given, limited set of choices. Constructed response items include those that require students to provide a text-based or numerical response.

Some selected response items, such as matching or multiple-selection items, have scoring guides to permit partial credit. Every constructed response item has a scoring guide that defines the criteria used to evaluate students' responses. Some short constructed response items can be scored according to guides that permit partial credit, while others are scored as either correct or incorrect. All constructed response scoring guides are refined from work with a sample of actual student responses gathered during item pilot testing. Students are provided information on elements required for a complete response in some of the discrete items and in overviews of composite items. This provides all students with greater access to the task and defines the parameters for their responses, honoring their time and energy as they engage in the work.

In 2026, the NAEP Mathematics Assessment retains selected and constructed response item types. The evolving capabilities of digital technology and the addition of NAEP Mathematical Practices mean the 2026 Framework includes the expansion of the two item types to allow for additional object-based and discourse/collaboration-based responses within discrete items and scenario-based tasks.

Selected Response

Selected response items for use on the NAEP Mathematics Assessment include a variety of formats.

- Single-selection multiple choice: Students respond by selecting a single choice from a set of given choices.
- Multiple-selection multiple choice: Students respond by selecting two or more choices that meet the condition stated in the stem of the item.
- Matching: Students respond by inserting (i.e., dragging and dropping) one or more source elements (e.g., a graphic) into target fields (e.g., a table).
- Zone: Students respond by selecting one or more regions on a graphic stimulus.
- Grid: Students evaluate mathematical statements or expressions with respect to certain properties. The answer is entered by selecting cells in a table in which rows typically correspond to the statements and columns to the properties checked.

- In-line choice: Students respond by selecting one option from one or more drop-down menus that may appear in various sections of an item.
- Conversational responses (new): Students respond by selecting from two or more choices of conversational responses as part of a discourse-based or collaborative task.

A new selected response item type included for the 2026 NAEP Mathematics Assessment involves the use of discourse and collaboration responses. Items of this type map most directly to the collaborative mathematics and modeling practices outlined in Chapter 3. Current examples ask a student to interact via a text-based scenario with avatars and choose (e.g., through multiple-choice, limited-option selections) from given conversational responses to move the collaborative problem forward. Such a selected response choice then provides some information about the level of collaborative mathematics the student exhibits.

Constructed Response

Constructed response items for the NAEP Mathematics Assessment also include a variety of formats, including those listed below.

- Short constructed response: Students respond by giving either a numerical result or the correct name or classification for a group of mathematical objects, or possibly by writing a brief explanation for a given result.
- Extended constructed response: Students respond by giving a description of a situation, an analysis of a graph or table of values or an algebraic expression, or a computation involving specific numerical values. These items require students to consider a situation that requires more than a numerical response or a short verbal communication.
- Object-based responses (new): Students respond by manipulating or using a physical object. The state of the object upon item completion is the response.

A new item type for NAEP Mathematics Assessments in 2026 and beyond is object-based responses. There is a growing ability to capture how students use manipulatives, both digital on-screen and with "smart" physical objects off-screen that can monitor activity and be connected to the digital assessment. Here there are at least two opportunities to be forward-thinking. First, further inquiry is warranted into ways to incorporate physical manipulatives that can collect data mapped to assessed constructs. The advances in smart tool technology are particularly suited to directly capture the NAEP Mathematical Practices outlined in Chapter 3. Second, further work is needed to align the data collected from tasks to valid measures of a construct. For example, one could imagine students manipulating a physical object, and the solution states that they come up with at different points in time (since activity is monitored continuously) could provide strong differentiating information about mathematical modeling. A solution state of the physical orientation of an object would be the answer (versus a discrete selection or clicking a multiple choice option). These and other opportunities will help NAEP move toward the ultimate goal of using tasks in the assessment in ways that capture the variety of ways students know and do mathematics.

Potential Scoring Advances

With the rapid advances in natural language processing, in the future there may be potential for mathematical collaboration to be assessed more effectively in open-ended constructed response formats. For example, the assessment might ask for and then automatically code responses where students are asked to explain their thinking or justify a contribution to collaborative mathematics.

While not available at the time of the 2026 Framework revision, such technology may become available for future administrations of the NAEP Mathematics Assessment and may increase accessibility. The assessment might ask students to input their thinking or dialogue via voice (with automatic transcription into text for coding and analysis), which would dramatically open up ways for students to demonstrate what they know and can do. Similarly, pairs of students might be asked to turn on an audio documentation (e.g., a recording device) as they work together on a modeling task. The record of discourse would be part of assessment response, measurable evidence of students creating representations, making conjectures, critiquing and debating, revoicing, or justifying their solutions to one another. Considerable research and development work are needed around the technology for natural language processing and related domains, combined with careful mapping to constructs and measurement needs, to realize the aspirational goal of opening up such ways for students to show what they can do mathematically. Also, special attention must be paid to issues of consent and privacy when considering voice recording.

Response Data and Process Data for Future NAEP Mathematics Assessments

A key challenge is the need to capture enough information about mathematics content and practices for a reliable and valid assessment. When this happens, within the context of scenario-based tasks, which require more time for engagement and completion, data may be available from fewer items per student.

An opportunity for future NAEP Mathematics Assessments is to develop validated measures from process data, which is generated based on student interaction with the tools and systems in the scenario-based tasks (e.g., clickstreams or activity logs). The data are different from what might be generated in a non-digital format, so it is necessary to describe how the additional data might be handled.

Conventional items always involve the student in a direct response, which generates *response data*. For example, after being presented with information in a table, the student is asked a text-based question and given a limited set of choices from which to select an answer. Student direct responses can also be used in scenarios. Direct response data can include selection from a set of choices (e.g., multiple choice, checking all boxes that apply, or providing a constructed response). Scoring methods for such response data are well established.

By contrast, *process data* reflect interactions in which the student engages in and may provide relevant evidence about whether the student possesses a skill that is an assessment target. Thus, process data can be captured, measured, and interpreted to generate a score. Clickstream data, activity logs, text, and transcribed voice responses are among the ways to capture the state of student activity as they work through a problem. These types of data hold potential power to measure student interactivity in modeling and collaborative mathematics, as well as levels of any mathematical practice (e.g., capturing frequency, density, and intensity of engagement with a mathematical practice or identifying and comparing novice to expert levels of a practice through process data). While this capability is powerful in theory, moving from big data sources to carefully constructed and validated measures is difficult to achieve in practice. A special study in the area of mathematics assessment is needed to explore and fully realize the potential of process data within digital scenario-based tasks.

NAEP Mathematics Tools

The preceding sections provide an overview for thinking through—and developing—diverse ways to show what students know and can do mathematically. Each response type requires related system tools and, at times, mathematics tools. In a digitally based environment, for example, students will require tools to enter mathematical expressions; to draw, highlight, and erase on the screen; to measure the lengths of virtual objects; to plot points on number lines or in coordinate planes; to graph lines and functions; and to create and modify graphical representations. Additionally, the testing environment will need to provide computational tools equivalent to a four-function calculator at grade 4, a scientific calculator at grade 8, and a graphing calculator at grade 12. Continuing a practice that began with the 2017 NAEP Mathematics Assessment, before the assessment, students complete a brief interactive tutorial designed to orient them to the mathematics tools they will use during the assessment. The 2019 tutorials for each grade level can be found on the Internet (Governing Board, 2019a, 2019b).

The digitally based environment of the 2026 NAEP Mathematics Assessment provides the majority of these mathematics tools digitally. All digital NAEP assessments include system tools, which are always available and common across all NAEP assessments. There are also mathematics tools, which are specific to and only available for certain items on NAEP Mathematics Assessments. The materials and accompanying tasks need to be carefully chosen to cause minimal disruption of the administration process, and would typically only be provided when relevant to solving the item. Continuing the calculator policy established for the 2017 digital administration, students will have access to a calculator emulator in blocks of items designated as “calculator blocks.” New in 2026 will be the availability of a graphing emulator for grade 12, since high school students typically use graphing calculators or online emulators and not scientific calculators (Crowe & Ma, 2010).

Examples of future digital mathematics tools for the 2026 NAEP Mathematics Assessment may include number tiles, spreadsheets, symbolic algebra manipulators, graphing tools, simulations, and dynamic geometry software. Continued development of mathematics tools (digital, physical, and other) can serve to achieve the goals of more authentic tasks for students and more diverse ways for students to demonstrate their knowledge and skills. Tools can allow for formal mathematics representations and symbols, and they can also allow students to create and share their own ways of thinking with their own representations. For example, some statistical tools allow students to construct their own graphical representations of data and create their own probability simulators. Considering what tools are needed for new items and the time it will take students to use them is an integral part of the assessment design process.

Accessibility

The NAEP Mathematics Assessment is designed to measure student achievement across the nation. Consequently, NAEP incorporates inclusive policies and practices into every aspect of the assessment, including selection of students, participation in the assessment administration, and valid and effective accommodations. NAEP is administered to a sample of students who represent the student population of the nation, regardless of race/ethnicity, socioeconomic status, disability, status as an English language learner, or any other factors. Similarly, for state-level results and results for the NAEP Trial Urban District Assessment, NAEP is administered to a sample of students who represent the jurisdiction. Therefore, the NAEP Mathematics

Assessment provides an opportunity for participating students to demonstrate mathematical knowledge and skill, including students who have learned mathematics in a variety of ways, followed different curricula, and used different instructional materials; students who have mastered mathematics content and practices to varying degrees; students with a variety of disabilities; and students who are English language learners. The related design issue is the development of a large-scale assessment that measures mathematics achievement of students who come to the assessment with different experiences, strengths, and challenges; who approach mathematics from different perspectives; and who have different ways of displaying their knowledge and skill.

NAEP uses two methods to design an accessible assessment program that provides accommodations for students with special needs. The first is addressed by careful item and delivery design with the full consideration of the range of participating students. For many students with disabilities and students whose native language is not English, the standard administration of the NAEP assessment will be most appropriate. For other students with disabilities (SD students) and some English language learners (ELL students), NAEP allows for a variety of accommodations, which can be used alone or in combination.

Some accommodations are built-in features, called Universal Design Elements, of the NAEP system tools that are available to all students. Other accommodations, such as additional assessment time, are offered for specific eligible students. Available accommodations fall into four categories (see Governing Board, n.d., specific information about accommodations):

- Standard NAEP Practice, available in almost all NAEP assessments for SD and ELL students.
- Other accommodations for SD students that require special presentation, such as Braille or sign language.
- Other accommodations for ELL students.
- Universal Design Elements that are built-in features of the computer-based assessments available to all students.

For more detailed information about accommodations, see the Governing Board's *NAEP Testing and Reporting of Students with Disabilities and English Language Learners Policy Statement* (2014a).

Matrix Sampling

The design of NAEP uses matrix sampling to enable a broad and deep assessment of students' mathematical knowledge and skill that also minimizes the time burden on schools and students. Matrix sampling is a sampling plan in which different samples of students take different samples of items. Students taking part in the assessment do not all receive the same items. Matrix sampling greatly increases the capacity to obtain information across a much broader range of the objectives than would otherwise be possible.

Balance of the Assessment

As mentioned earlier, the goal is to create an authentic assessment, one based on the experiences of students that will diversify the ways that students can show what they know and can do in mathematics. The emphasis placed on NAEP Mathematical Practices in this framework increases interdependence since multiple practices may be assessed simultaneously in the context of one item. The expansion of item types to include scenario-based tasks also complicates the assessment design.

The balance of content and practices having been introduced in Chapters 2 and 3, respectively, a summary of all three balance dimensions follows.

- Balance by Mathematics Content
    - Number Properties and Operations
    - Measurement
    - Geometry
    - Data Analysis, Statistics, and Probability
    - Algebra
- Balance by Mathematical Practice
    - Representing
    - Abstracting and Generalizing
    - Justifying and Proving
    - Mathematical Modeling
    - Collaborative Mathematics
- Balance by Response Type
    - Selected response
    - Constructed response (short and extended)

Balance of Mathematics Content

Each NAEP Mathematics Assessment item or item part is developed to measure one content objective. Exhibit 4.2 reproduces the distribution of items by grade and content area (from Exhibit 2.1). See Chapter 2 for further details.

Exhibit 4.2. Percentage Distribution of Items by Grade and Content Area

<table>
<tr><th>Content Area</th><th>Grade 4</th><th>Grade 8</th><th>Grade 12</th></tr>
<tr><td>Number Properties and Operations</td><td>45*</td><td>20</td><td>10</td></tr>
<tr><td>Measurement</td><td>20</td><td>10</td><td rowspan="2">30</td></tr>
<tr><td>Geometry</td><td>15</td><td>20</td></tr>
<tr><td>Data Analysis, Statistics, and Probability</td><td>5</td><td>20</td><td>25</td></tr>
<tr><td>Algebra</td><td>15</td><td>30</td><td>35</td></tr>
</table>

* Note: At least one-third of grade 4 Number Properties and Operations items should assess fraction content.

Balance of Mathematical Practices

The target percentage ranges of items for each NAEP Mathematical Practice are reproduced in Exhibit 4.3 (from Exhibit 3.24). Most NAEP Mathematics Assessment items will feature one of the five NAEP Mathematical Practices (55 to 85 percent). The balance of items (15 to 45 percent), those in the "Other" category, will assess knowledge of content without calling on a particular NAEP Mathematical Practice. Because of the matrix sampling used on the NAEP Mathematics Assessment, the proportions in Exhibit 4.3 are for the entire pool of items used and do not represent the experience of each student. See Chapter 3 for further details about the NAEP Mathematical Practices.

Exhibit 4.3. Percentage Distribution of Items by NAEP Mathematical Practice

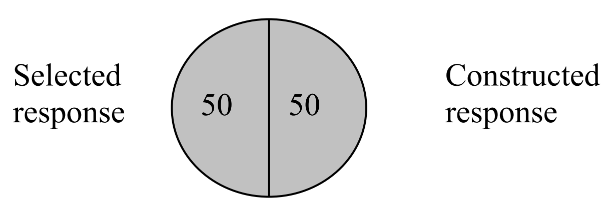
NAEP Mathematical Practice Area	Percentage of Items
Representing	10–15
Abstracting and Generalizing	10–15
Justifying and Proving	15–25
Mathematical Modeling	10–15
Collaborative Mathematics	10–15
Other	15–45

Certain formats are likely to be especially valuable in eliciting particular NAEP Mathematical Practices. As illustrated in Chapter 3, discrete items are useful measures of NAEP Mathematical Practices such as Representing, Abstracting and Generalizing, and Justifying and Proving. Also, as noted in Chapter 3, Mathematical Modeling and Collaborative Mathematics are more appropriately measured by scenario-based tasks.

Balance by Response Type

Items include selected response and constructed response types, and these response types may also occur within scenario-based tasks. Selected response includes traditional single-selection multiple choice, as well as other response types such as matching, zone, in-line choice, grid, and limited option responses. These items are machine scored. Constructed response includes short and extended constructed response. Constructed response items may include item types such as fill-in-the-blank, extended text, digital tool–based, and object-based constructed responses, as well as discourse and collaboration responses. Testing time on NAEP is divided evenly between selected response items and constructed response items, as shown in Exhibit 4.4.

Exhibit 4.4. Percent of Testing Time by Response Type



REPORTING RESULTS OF THE NAEP MATHEMATICS ASSESSMENT

NAEP provides the nation with a snapshot of what U.S. students know and can do in mathematics. Results of the NAEP Mathematics Assessment administrations are reported in terms of average scores for groups of students on the NAEP 0–500 scale and as percentages of students who attain each of the three achievement levels (*NAEP Basic*, *NAEP Proficient*, and *NAEP Advanced*). This is an assessment of overall achievement, not a tool for diagnosing the needs of individuals or groups of students. Reported scores are always at the aggregate level; by law, scores are not produced for individual schools or students. Results are reported for the nation as a whole, for regions of the nation, for states, and for large districts that volunteer to participate in the NAEP Trial Urban District Assessment (TUDA). The NAEP results are published in an interactive version online as *The Nation's Report Card* (Governing Board, n.d.). The online resource provides detailed information on the nature of the assessment, the demographics of the students who participate, the assessment results, and the contexts in which students are learning.

Legislative Provisions for NAEP Reporting

Under the provisions of the Every Student Succeeds Act (ESSA), states receiving Title I grants must include assurance in their state plans that they will participate in the reading and mathematics state NAEP at grades 4 and 8. Local districts that receive Title I funds must agree to participate in biennial NAEP reading and mathematics administrations at grades 4 and 8 if they are selected to do so as part of the NAEP sample. Their results are included in state and national reporting. Participation in NAEP will not substitute for the mandated state-level assessments in reading and mathematics at grades 3 to 8. An important development over the last 20 years has been an evolving understanding of how NAEP complements state assessments, which are tightly aligned with state standards.

In 2002, NAEP initiated TUDA in five large urban school districts that are members of the Council of the Great City Schools (the Atlanta City, City of Chicago, Houston Independent, Los Angeles Unified, and New York City Public Schools districts). In 2003, additional large urban districts began to participate in these assessments, growing to a total of 27 districts by 2017. TUDA is administered biennially in odd-numbered years in tandem with NAEP state-level assessments. Sampled students in TUDA districts are assessed in the same subjects and use the same NAEP field materials as students selected as part of national main or state samples. TUDA results are reported separately from the state in which the TUDA is located, but results are not reported for individual students or schools. With student performance results reported by district, participating TUDA districts can use results for evaluating their achievement trends and for comparative purposes. Here too the complementarity of NAEP with state and local assessments is important to support so as to avoid unnecessary additional testing and to maximize useful information for educators and policymakers to use.

Reporting Scale Scores and Achievement Levels

The NAEP Mathematics Assessment is reported in terms of percentages of students who attain each of the three achievement levels: *NAEP Basic*, *NAEP Proficient*, and *NAEP Advanced*.

Reported scores are always at the aggregate level. This framework calls for NAEP results to continue to be reported in terms of sub-scores as well, for each content area. Cut scores represent the minimum score required for performance at each NAEP achievement level. Cut scores are reported along with the percentage of students who scored at or above the cut score.

This framework calls for reporting on NAEP Mathematical Practices. Since these practices are fundamentally intertwined with NAEP mathematics content areas, there will not be separate reporting scales for each NAEP Mathematical Practice. Options for measuring and reporting on NAEP Mathematical Practices are described in the *Assessment and Item Specifications* document.

Reporting on achievement levels is one way in which NAEP results reach the general public and policymakers. Since 1990, the Governing Board has used achievement levels for reporting results on NAEP assessments; achievement level results indicate the degree to which student performance meets the standards set for what students should know and be able to do at the *NAEP Basic, NAEP Proficient,* and *NAEP Advanced* levels. Descriptions of achievement levels articulate expectations of performance at each grade level (see Exhibit 5.1). They are reported as percentages of students within each achievement level range, as well as the percentage of students at or above *NAEP Basic* and at or above *NAEP Proficient* ranges. Students performing at or above the *NAEP Proficient* level on NAEP assessments demonstrate solid academic performance and competency over challenging subject matter.

It should be noted that the *NAEP Proficient* achievement level does not represent grade-level proficiency as determined by other assessment standards (e.g., state or district assessments) and there are significant differences between achievement in the context of NAEP as compared to the context of state-level annual tests. For one, teachers and students are not expected to have studied the NAEP framework or systematically aligned state standards or local curricula with it, nor are students expected to study intensively for the assessment. Furthermore, the NAEP assessment is broader than a typical state grade-level test, for NAEP covers multiple years of study and does not focus on specific instructional units and school years.

Results for students not reaching the *NAEP Basic* achievement level are reported as below *NAEP Basic*. As noted, individual student performance cannot be reported based on NAEP results.

NAEP Achievement Level Descriptions

Since 1990, the Governing Board has used achievement levels for reporting results on NAEP assessments. The achievement levels represent an informed judgment of "how good is good enough" in the various subjects that are assessed. Generic policy definitions for achievement at the *NAEP Basic, NAEP Proficient,* and *NAEP Advanced* levels describe in very general terms what students at each grade level should know and be able to do on the assessment. Achievement level descriptions specific to the 2026 NAEP Mathematics Framework can be found in Appendix A. These will be used to guide item development and initial stages of standard setting for the 2026 NAEP Mathematics Assessment, if it is necessary to conduct a new standard setting.

The content achievement level descriptions may be revised for achievement level setting, if additional information is obtained or required. A broadly representative panel of exceptional teachers, educators, and professionals in mathematics will be convened to engage in a standard-setting process to determine cut scores that correspond to the achievement level descriptions. All achievement level setting activities for NAEP are performed in accordance with current best practices in standard setting and the Governing Board's *Developing Student Achievement Levels for the National Assessment of Educational Progress Policy Statement* (2018a). The Governing Board policy does not extend to creating achievement level descriptions for performance below the *NAEP Basic* level.

Exhibit 5.1. Generic Achievement Level Policy Definitions for NAEP

Achievement Level	Definition
NAEP Advanced	This level signifies superior performance beyond *NAEP Proficient.*
NAEP Proficient	This level represents solid academic performance for each NAEP assessment. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
NAEP Basic	This level denotes partial mastery of prerequisite knowledge and skills that are fundamental for performance at the *NAEP Proficient* level.

Contextual Variables

NAEP law (Governing Board, 2017b) requires reporting according to various student populations (see section 303[b][2][G]), including:

a. Gender,
b. Race/ethnicity,
c. Eligibility for free/reduced-price lunch,
d. Students with disabilities, and
e. English language learners.

At times, people presume that the categories used to report data are related to causal explanations for observed differences, for example, that gender accounts for performance. Although differences in student achievement are often referred to as "achievement gaps," scholars have long found that these differences also represent gaps in students' opportunities to learn (e.g., Carter & Welner, 2013; Flores, 2007; Martin, 2009; Schmidt et al., 2015), as discussed in Chapter 1. When results are interpreted in ways that emphasize achievement gaps without attending to opportunity gaps, score differences across subgroups of students can be misinterpreted as differences in student ability, rather than differences due to unequal and inadequate educational opportunities.

The *Standards for Educational and Psychological Testing* (AERA, APA, & NCME, 2014) recommend that reports of group differences in assessment performance be accompanied by relevant contextual information, where possible, to both discourage erroneous interpretation and enable meaningful analysis of the differences. That standard reads as follows:

> Reports of group differences in test performance should be accompanied by relevant contextual information, where possible, to enable meaningful interpretation of the differences. If appropriate contextual information is not available, users should be cautioned against misinterpretation. (Standard 13.6)

Contextual data about students, teachers, and schools are needed to fulfill the statutory requirement that NAEP include information, whenever feasible, for these groups which promotes meaningful interpretation. The important components of NAEP reporting are summarized in Exhibit 5.2.

Exhibit 5.2. Components of NAEP Reporting

Component	Key Characteristics
How Information Is Reported	Elements released to the public include: • Results published mainly online with an interactive report card • Performance of various subgroups at the national level, published online • Online data tools with sample questions, performance associated with all collected contextual variables, item maps, and profiles of states and TUDA districts
What Is Reported	NAEP data are reported by: • Percentage of students attaining achievement levels • Scale scores • Sample responses to illustrate achievement level definitions • Contextual information from NAEP questionnaires

Contextual variables are selected to be of topical interest, timely, and directly related to academic achievement and current trends and issues in mathematics. In the past, a range of information has been collected as part of NAEP. In one analysis, Pellegrino, Jones, and Mitchell (1999) identified five existing categories of indicators: (1) student background characteristics; (2) home and community support for learning; (3) instructional practices and learning resources; (4) teacher education and professional development; and (5) school climate.

Contextual variables for the 2026 NAEP Mathematics Assessment will build on two broad categories: student factors and opportunity to learn factors. Student factors have been described as skills, strategies, attitudes, and behaviors that are distinct from content knowledge and academic skills. Opportunity to learn factors have been described as whether students are exposed to opportunities to acquire relevant knowledge and skill in or out of school. These are described in the following section.

Mathematics-Specific Contextual Variables

As noted in Chapter 1, research has informed an expanded view of the factors that shape opportunities to learn, including time, content and practices, instructional strategies (e.g., how students are grouped for learning; the mathematical tasks they engage in; the opportunities students have to reason, model, and debate ideas), and instructional resources (e.g., human, material, and social resources that shape student access to mathematics).

For example, research has demonstrated that what students learn is shaped by the availability of various mathematics programs, curricula, extracurricular activities geared toward mathematics, the percentage of teachers certified in mathematics, teacher years of experience, percentage of mathematics teachers on an emergency license or vacancies/substitute teachers in the school, and number of teachers with mathematics degrees, among other factors. Teachers' and administrators' beliefs about what mathematics is, how one learns mathematics, and who can learn mathematics also affect student learning. What students learn is shaped by their sense of identity and agency. Students who see themselves, and who are seen by others, as capable mathematical thinkers are more likely to participate in ways that further their learning; students who do not see themselves, and are not seen by others, as capable mathematical thinkers are likely to be disengaged. Steele, Spencer, and Aronson (2002), for example, found that even passing reminders that a student is a member of one group or another—often, in this case, a group that is stereotyped as intellectually or academically inferior—can undermine student performance.

There are countless factors that shape what and when students learn. The NAEP Mathematics student, teacher, and administrator surveys cannot possibly cover all such factors. Even though it would be helpful to ask students and teachers the same questions, that too is not possible given time constraints. Furthermore, questions about some factors may not be appropriate in the NAEP context. Given the constraints, not all topics can be addressed.

To support prioritization and ensure that NAEP results have appropriate context for interpretation, this framework sets the following topics to receive the greatest emphasis in the 2026 NAEP Mathematics Assessment's contextual questionnaires (in order of priority).

- *Mathematics content and practices*. The 2026 NAEP Mathematics Framework conceptualizes mathematics as both content and practices. Therefore, contextual variables related to mathematics content are expanded to include reference to NAEP Mathematical Practices as well. Interpreting students' achievement requires a basic understanding of what mathematics content and practices students have engaged with. Given variation across states in standards and frameworks, this information is crucial.
- *Teacher factors*. Research demonstrates that teacher quality is a critical in-school factor in predicting student achievement. This framework prioritizes the collection of data on teacher preparation and professional development, as well as teacher mathematical knowledge for teaching.
- *Student mathematical identity*. Research demonstrates that students' perceptions of their mathematical identity directly relates to their mathematics learning. This framework prioritizes gathering information about students' mathematical identities through questions that address student participation in activities such as discussion of mathematical ideas or evaluation of how a mathematics problem is framed.

- *Instructional resources*. A range of resources influences instruction, including school climate, instructional leadership, additional instructional personnel, time, technology, curriculum, and materials. This framework prioritizes gathering information about school resources that can inform the interpretation of results, including students' exposure to different types of technology, the time devoted to mathematics teaching and learning in school, and the curricular and instructional materials at teachers' and students' disposal to support learning. In terms of technology, questionnaires will emphasize what technology is *available* to support mathematics teaching and learning.
- *Instructional organization and strategies*. Interpreting student achievement levels will also depend on understanding the instructional strategies used in mathematics class, including collaborating in small-group work, engaging in mathematical discussions, and using a range of tools to represent and model mathematics. This framework prioritizes gathering information both on the organization of classrooms and on the instructional routines and approaches that teachers use. It also includes what technologies and formative assessments are *used* in instruction.

Conclusion

As the Nation's Report Card, NAEP reports on student achievement over time, presenting an analysis of national trends in students' mathematical competence. The NAEP Mathematics Assessment is designed to assess the achievement of groups of students through robust and challenging assessments that are well aligned with current understanding of the mathematics content and practices to be learned and that use technology in ways that maximize both student engagement and accessibility. The results of the assessment are informed by data on contextual variables that illuminate potential differences in opportunities to learn for students.

Based on current research, policy, and practice, the NAEP Mathematics Framework visioning and development process articulated several major goals: to expand attention to student engagement in reasoning about and doing mathematics, to adjust NAEP's mathematics domains and competencies, to leverage interactive multimedia scenario-based tasks as a way to provide more authentic tasks for students to complete and to increase the assessment's accessibility, and to develop an expansive conception of opportunities to learn that would inform the collection and use of contextual information. Accordingly, Chapters 2 and 3 describe the content and practices of mathematics on which students should be measured on the 2026 NAEP Mathematics Assessment as the Nation's Report Card. Chapter 4 describes the expansion of the assessment in ways that prudently leverage technology's potential to increase authenticity and accessibility. Chapters 1 and 5 describe an expansive understanding of opportunities to learn, and the role that contextual information plays in meaningful interpretation of the results from future NAEP Mathematics Assessments based on this framework.

The ultimate goal of our nation's schools is to ensure that every student has access to learning high-quality mathematics. NAEP plays an important role in providing a broad picture of students' knowledge and skills in mathematics to the nation. NAEP scores, illuminated by relevant contextual information, can provide the public, families, students, and schools useful data on student performance that complements information provided by state tests that are more tightly aligned with specific state standards. As a view of present trends, it provides invaluable data to inform policy and practice in the future.

Abstracting and Generalizing: A NAEP Mathematical Practice involving decontextualizing; identifying commonality across cases, items, problems, or representations; and extending one's reasoning to a broader domain appropriate for the grade level and the mathematics being assessed.

Achievement level descriptions (ALDs): Descriptions of student performance at achievement levels (basic, proficient, and advanced), detailing what students should know and be able to do in terms of the mathematics content areas and practices on the NAEP assessment.

Clickstream: Response and process data generated based on student interactions with tools and systems in scenario-based tasks.

Cognitive complexity: The state or quality of a thought process that involves numerous constructs, with many interrelationships among them. Such mental processing is often experienced as difficult or effortful.

Collaborative Mathematics: A NAEP Mathematical Practice that involves the social enterprise of doing mathematics with others through discussion and collaborative problem solving whereby ideas are offered, debated, connected, and built-upon toward solution and shared understanding. Collaborative mathematics involves joint thinking among individuals toward the construction of a problem solution.

Construct: An image, idea, or theory, especially a complex one formed from a number of simpler elements, and often embedded in a web of related ideas.

Constructed response: An open-ended, text-based response. Every constructed response item has a scoring guide that defines the criteria used to evaluate students' responses.

Context: The physical, temporal, historical, cultural, or linguistic setting for an event, performance, statement, or idea, and in terms of which such events or statements can be fully understood and assessed.

Contextual variables: Student, teacher, administrator, and school factors that shape students' opportunities to learn, including time, content, instructional strategies, and instructional resources.

Conversational response: A response within a discourse-based or collaborative task in which students respond by selecting from two or more choices that reflect a conversation between characters described in the task.

Deduction: Reasoning that makes a logical argument, draws conclusions, and applies generalizations to specific situations.

Discourse: Denotes written and spoken communications or “language-in-use” (Gee, 1999). Discourse can also refer to the totality of codified language used in a given field of intellectual enquiry and of social practice.

Discrete items: Stand-alone assessment items.

English language learner: Active learners of the English language who may benefit from various types of language support programs; students from a diverse set of backgrounds who often come from non-English-speaking homes and backgrounds, and who typically require specialized or modified instruction in both the English language and in their academic courses.

Funds of knowledge: The strengths students bring with them to the classroom, including academic and personal background knowledge, accumulated life experiences, skills and knowledge used to navigate everyday social contexts, and world views structured by broader historically and politically influenced social forces (Civil, 2016; González, et. al, 2005).

GAIMME: *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (Garfunkel & Montgomery, 2019). A report issued by a collaboration between the Society for Industrial and Applied Mathematics and the Consortium for Mathematics and Its Applications.

Generalization: The act of identifying a property that holds for a larger set of mathematical objects or conditions than the number of individually verified cases.

Induction: Reasoning that begins with specific observations to develop generalizations and conclusions; looking for patterns and making generalizations.

In-line choice items: Items in which students respond by selecting one option from one or more drop-down menus that may appear in various sections of an item.

Instructional practice: Teaching methods that guide interaction in the classroom.

Joint thinking: Working and thinking together on a shared goal, including sharing ideas with others; attending to and making sense of the mathematical contributions of others; evaluating the merit of others’ ideas through agreement or disagreement; and productively responding to others’ ideas through building on or extending ideas and connecting or generalizing across ideas.

Justifying and Proving: A NAEP Mathematical Practice that involves creating, evaluating, showing, or refuting mathematical claims in developmentally and mathematically appropriate ways.

Mathematical argumentation: The action or process of reasoning systematically in support of an idea, action, or theory.

Mathematical justification: A critical aspect of the NAEP Mathematical Practice of Justifying and Proving that includes creating arguments, explaining why conjectures must be true or demonstrating that they are false, exploring special cases or searching for counterexamples, understanding the role of definitions and counterexamples, and evaluating arguments.

Mathematical knowledge for teaching: The specialized knowledge mathematics teachers need to support their students' learning that goes beyond the mathematics that any educated adult might need; the mathematics-specific knowledge of content, pedagogy, and students that is needed to perform the recurrent tasks of teaching mathematics to students (Ball, Thames, & Phelps, 2008).

Mathematical literacy: The application of numerical, spatial, or symbolic mathematical information to situations in a person's life as a community member, citizen, worker, or consumer.

Mathematical Modeling: A NAEP Mathematical Practice that involves making sense of a scenario, identifying a problem to be solved, mathematizing it, applying the mathematization to reach a solution, and checking the viability of the solution.

Mathematical practice: The working methods of *doing* mathematics, including the NAEP Mathematical Practices of Representing, Abstracting and Generalizing, Justifying and Proving, Mathematical Modeling, and Collaborative Mathematics.

Mathematical proof: A formal proof is a specific type of argument "consisting of logically rigorous deductions of conclusions from hypotheses" (NCTM, 2000, p. 55). The form used to represent a mathematical proof is valid as long as it communicates the essential features of the proof; that is, it contains logically connected mathematical statements that are based on valid definitions and theorems.

Mathematical problem solving: Completing mathematical tasks where the task contexts may range from the purely mathematical to those that are experientially concrete or real to students.

Mathematical reasoning: A skill that involves using other mathematical skills, including evaluating situations, selecting problem-solving strategies, drawing logical conclusions, developing and describing solutions, and recognizing how those solutions can be applied. Mathematical reasoners are able to reflect on solutions to problems and determine whether or not they make sense.

Object-based responses: Assessment responses that involve manipulating or using a physical object.

Opportunity gap: Relates to the inputs, the unequal or inequitable distribution of resources and opportunities, that contribute to and perpetuate lower educational achievement and attainment based on race, ethnicity, socioeconomic status, English proficiency, community wealth, familial situations, or other factors.

Opportunity to learn: Inputs and processes that enable student achievement of intended outcomes.

PISA: The Programme for International Student Assessment, an international assessment that measures 15-year-old students' reading, mathematics, and science literacy every three years.

Representing: A NAEP Mathematical Practice that involves recognizing, using, creating, interpreting, or translating among representations appropriate for the grade level and the mathematics being assessed.

Revoicing: A method of communication that can be used by students or teachers to "re-utter another's contribution through the use of repetition, expansion, or rephrasing" (Enyedy et al., 2008, p. 135).

Scenario-based task: Assessment tasks that have both context and extended storylines to provide opportunities to demonstrate facility with NAEP Mathematical Practices.

Selected response: Assessment responses that involve a student selecting one or more response options from a given, limited set of choices.

Single-selection multiple choice: Assessment items in which students respond by selecting a single choice from a set of given choices.

Student identity: A person's evolving view of self in a given social context influenced by their experiences, personal history, and other events. Students' mathematical identity is how they see themselves in relation to mathematics and mathematics learning (Bishop, 2012).

Tool-based responses: Assessment responses that involve manipulating or using a virtual tool on-screen (e.g., an on-screen ruler).

APPENDIX A: NAEP MATHEMATICS ACHIEVEMENT LEVEL DESCRIPTIONS

The NAEP Achievement Level Descriptions (ALDs) in this appendix provide examples of what students performing at the *NAEP Basic*, *NAEP Proficient*, and *NAEP Advanced* achievement levels should know and be able to do in terms of the mathematics content areas and practices identified in this framework. The intended audiences for these ALDs are the NAEP assessment development contractor and item writers; the ALDs help ensure that a broad range of items is developed at each assessed grade.

The ALDs in the 2026 NAEP Mathematics Framework have changed, relative to ALDs presented in the previous frameworks. The differences reflect not only changes to the mathematics knowledge, skills, and abilities assessed (mathematics content areas and mathematical practices) but also an effort to develop ALDs that provide explicit guidance for item developers. Specifically, across grade levels, the 2026 Framework ALDs have changed in the following ways:

- Updates to the grade-level objectives in Chapter 2 of this framework are reflected in the content foci described in each grade-level ALD.
- Mathematical Practices are new to the 2026 Framework and are made explicit at every achievement level in every grade in these ALDs. The mathematical practices absorbed much of the reasoning and problem-solving language from previous framework ALDs. As noted in Chapter 3, some NAEP Mathematics items will not assess a NAEP Mathematical Practice. Thus, some elements of the NAEP Mathematics ALDs are not linked to a NAEP Mathematical Practice. Instead, they are associated with other activities such as enacting knowledge of mathematical facts, using procedural fluency, and engaging in mathematical practices that are not included in the five identified for the NAEP Mathematics Assessment.
- Although Chapter 4 of this framework provides examples of digital tools (e.g., graphing tools) that may be common in 2026 and beyond in schools, these ALDs have reduced the focus on technology-specific descriptions of the mathematics students should know and be able to do on the NAEP Mathematics Assessment.
- To provide specific and unambiguous guidance to item developers, these ALDs provide more explicit elaborations of the knowledge and skills students should demonstrate and the actions they should perform at each grade level and within each achievement level.

To add clarity and specificity, the ALDs in this framework include example items targeting each achievement level within each grade level. Following the ALDs presentation, in Appendix B, three sets of items (one set each for grades 4, 8, and 12) illustrate the knowledge and skills required at different NAEP achievement levels. The items are not intended to represent the entire set of mathematics content areas or practices, nor do the items imply priority or importance of some content areas or practices above others.

Finally, to guard against misinterpretations, it is important to clarify the intended meaning of the term *routine*, which is used frequently in the ALDs. For the purposes of the ALDs, *routine* is defined as *having a readily available solution method*.

Mathematics Achievement Level Descriptions for Grade 4

NAEP Basic	**Grade 4 students performing at the *NAEP Basic* level should show evidence of emergent understanding of mathematics concepts and procedures in the five NAEP content areas. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 4 students performing at the *NAEP Basic* level should be able to estimate and perform paper and pencil computations with whole numbers (e.g., addition and subtraction within 1,000; multiplication and division within 100); understand the meaning of fractions and decimals, but not necessarily the relations between fractions and decimals; compare numbers to familiar benchmarks such as 0, ¼, ½, ⅔, ¾, and 1; identify or measure attributes of simple plane figures (e.g., triangles, rectangles, squares, and circles) and simple solid figures (e.g., cubes, spheres, and cylinders), choosing appropriate measuring tools and units of measure; and solve problems involving these concepts and procedures. Students should be able to represent whole numbers, fractions, and decimals using visual representations; draw or sketch simple plane figures from a written description; create a visual, graphical, or tabular representation of a given set of data; and recognize, describe (in words or symbols), or extend numerical and visual patterns. They should be able to explain or defend strategies or solutions (e.g., justify solutions to word problems through numeric representations and operations); make mathematical sense of a problem scenario; select and use visual, physical, or symbolic representations, as needed, to lead to solutions; and share ideas and revoice the ideas of others.
NAEP Proficient	**Grade 4 students performing at the *NAEP Proficient* level should be able to recognize when particular concepts, procedures, and strategies are appropriate, and select, integrate, and apply them to represent or model situations mathematically and solve problems requiring more than the application of a known procedure or strategy. Students should be able to reason about relationships involving the domains of number, space, or data. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 4 students performing at the *NAEP Proficient* level should be able to estimate and compute with whole numbers (within the guidelines set by the NAEP objectives) and determine whether and explain why the results are reasonable; identify, represent, compare, add, and subtract fractions and decimals, using visual representations to compare numbers and as tools to solve problems; identify or draw angles; draw or sketch simple plane and solid figures from a written description; read and interpret a single set of data, including the interpretation of graphical or tabular representations of data; extend their understanding of patterns to create a different

	representation of a pattern or sequence; and create, use, and defend visual representations of problem situations involving these concepts and procedures. In all content areas, students should be able to abstract or de-contextualize and re-contextualize ideas in routine problems using written and symbolic structures; create and evaluate mathematical arguments; explain why conjectures must be true or demonstrate that they are false; explore with examples or search for counterexamples and understand the role of counterexamples in mathematical arguments; determine assumptions, pose answerable questions, and determine tools to use as they interpret and solve problems; and make sense of and evaluate the mathematical contributions of others through expressing and defending agreement or disagreement.
NAEP Advanced	**Grade 4 students performing at the *NAEP Advanced* level should be able to apply conceptual understanding and procedural knowledge in non-algorithmic ways to complex and non-routine mathematical or real-world problems in the five NAEP content areas. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 4 students performing at the *NAEP Advanced* level should be able to solve complex and non-routine real-world problems in all NAEP content areas. These students should be able to draw logical conclusions from the results of a solution process; justify answers and solution processes by explaining how and why they were achieved; and use words or symbols to generalize a pattern appearing in a sequence or table. Students should be able to build on, analyze, and justify representations or mathematical models created by others; use structures and patterns to generate a rule and investigate conditions under which the rule applies; use a variety of grade-appropriate methods to justify or refute a mathematical statement using valid definitions, statements, or counterexamples; determine and use a series of processes to mathematize a complex or non-routine situation and evaluate the results obtained; and extend, connect, or generalize across the ideas of others.

Mathematics Achievement Level Descriptions for Grade 8

NAEP Basic	**Grade 8 students performing at the *NAEP Basic* level should show evidence of emergent understanding, recognition, and application of concepts and procedures in the five NAEP content areas. Students should show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 8 students performing at the *NAEP Basic* level should be able to estimate and perform paper-and-pencil computations with rational numbers, including integers; solve linear equations or inequalities; choose appropriate measuring tools and units of measure; and solve problems involving strategic reasoning with these concepts and procedures, including using proportional reasoning to represent and solve routine problems. Students should be able to visually represent rational numbers, including decimals and integers, and use these representations as tools to solve problems; draw or sketch polygons, circles, or semicircles from a written description; create a visual, graphical, or tabular representation of a given set of data; and recognize, describe (in words or symbols), or extend numerical and visual patterns. They should be able to explain or defend strategies or solutions (e.g., justify solutions to word problems through numeric representations and operations); make mathematical sense of a problem scenario, selecting and using visual, physical, or symbolic representations, as needed, to lead to solutions; and share ideas and revoice the ideas of others.
NAEP Proficient	**Grade 8 students performing at the *NAEP Proficient* level should show evidence of recognizing and applying concepts and procedures to solve problems requiring more than routine application of a known process or result in the five NAEP content areas. They should recognize when particular concepts, procedures, and strategies are appropriate and select, integrate, and apply them to represent or model situations mathematically. Students should be able to reason about relationships involving the domains of number, space, or data. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 8 students performing at the *NAEP Proficient* level should understand the connections among integers, fractions, percents, and decimals and be able to work across these sets of numbers to examine proportional and linear relationships; expand their understanding of algebraic relationships to translate between different representations, compare properties of two relationships each represented differently, identify linear functions, and use the structure of an algebraic expression to solve problems; estimate the size of an object with respect to a given measurement attribute (e.g., length, area, volume, angle measurement, weight, or mass); compare figures or objects with respect to a measurement attribute; identify, describe, and justify relationships of congruence, similarity, and symmetry; organize data in order to make inferences and draw conclusions, interpret data in terms of generalized

	phenomena (e.g., shape, center, spread, clusters), and make comparisons or explore differences within and among sets of data; and interpret and apply probability concepts to routine situations. In all content areas, students should be able to abstract or de-contextualize and re-contextualize ideas in routine problems using written and symbolic structures; create and evaluate mathematical arguments; explain why conjectures must be true or demonstrate that they are false; explore with examples or search for counterexamples and understand the role of definitions and counterexamples in mathematical arguments; determine assumptions, pose answerable questions, and determine tools to use as they interpret and solve problems; and make sense of and evaluate the mathematical contributions of others through expressing and defending agreement or disagreement.
NAEP Advanced	**Grade 8 students performing at the *NAEP Advanced* level should be able to apply conceptual understanding and procedural knowledge in non-algorithmic ways to complex and non-routine mathematical or real-world problems. They should also be able to justify, generalize, and apply concepts and procedures, and be able to synthesize concepts and processes in the five NAEP content areas. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 8 students performing at the *NAEP Advanced* level should be able to solve complex and non-routine real-world problems in all NAEP content areas. They should be able to probe examples and counterexamples in order to shape generalizations from which they can develop mathematical models; use number sense and geometric awareness (e.g., definitions, properties of and relationships between geometric figures, results of transformations) to consider the reasonableness of an answer; and create problem-solving techniques, explaining the reasoning processes underlying their conclusions. Students should be able to use, analyze, and justify representations created by others; use structures and patterns to generate a rule and investigate conditions under which the rule applies; use a variety of grade-appropriate proof methods to justify a mathematical statement using valid definitions, statements, or counterexamples; determine and use a series of processes to mathematize a complex or non-routine situation and evaluate the results obtained; and extend, connect, or generalize across the ideas of others.

Mathematics Achievement Level Descriptions for Grade 12

NAEP Basic	**Grade 12 students performing at the *NAEP Basic* level should show evidence of emergent understanding, recognition, and application of concepts and procedures in the five NAEP content areas. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 12 students performing at the *NAEP Basic* level should be able to estimate and perform computations with real numbers, including irrational numbers; select appropriate units related to representing or measuring an attribute of an object; identify and describe relationships of congruence, similarity, and symmetry; organize data in order to make inferences and draw conclusions; interpret data in terms of generalized phenomena (e.g., shape, center, spread, clusters); make comparisons or explore differences within and among sets of data; interpret and apply probability concepts to routine situations; recognize, identify, and interpret information about functions presented in various forms; and solve problems involving these concepts and procedures, including using the coordinate plane to model and solve routine problems. Students should be able to represent real numbers, including very large and very small numbers, using visual representations and numerical expressions (e.g., scientific notation), and use these representations and expressions as tools to solve problems; draw or sketch plane figures and planar images of three-dimensional figures from a written description; create a visual, graphical, or tabular representation of a given set of data; and recognize, describe, or extend numerical patterns, including arithmetic and geometric progressions. They should be able to explain or defend strategies or solutions (e.g., justify solutions to word problems through numeric representations and operations); make mathematical sense of a problem scenario, selecting and using visual, physical, or symbolic representations, as needed, to lead to solutions; and share ideas and revoice the ideas of others.
NAEP Proficient	**Grade 12 students performing at the *NAEP Proficient* level should be able to recognize when particular concepts, procedures, and strategies are appropriate and to select, integrate, and apply them to represent or model situations mathematically to solve problems requiring more than the application of a known result. Students should be able to reason about relationships involving the domains of number, space, or data. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 12 students performing at the *NAEP Proficient* level should be able to solve complex non-routine items using algebraic and geometric approaches. Students should be able to find, test, and validate geometric and algebraic results and conjectures using a variety of methods. They should be able to

	design and carry out statistical surveys and experiments and interpret results that are obtained by them or by others. Students should also be able to translate between representations of functions (linear and nonlinear, quadratic and exponential), including verbal, graphical, tabular, and symbolic representations. In all content areas, students should be able to abstract or de-contextualize and re-contextualize ideas in routine problems using written and symbolic structures; create and evaluate mathematical arguments; explain why conjectures must be true or demonstrate that they are false; explore with examples or search for counterexamples and understand the role of definitions and counterexamples in mathematical arguments; determine assumptions, pose answerable questions, and determine tools to use as they interpret and solve problems; and make sense of and evaluate the mathematical contributions of others through expressing and defending agreement or disagreement.
NAEP Advanced	**Grade 12 students performing at the *NAEP Advanced* level should demonstrate in-depth knowledge of and be able to reason about mathematical concepts and procedures in the realms of number, algebra, geometry, and statistics. Students should also show evidence of engagement in the five NAEP Mathematical Practices as detailed.** Grade 12 students performing at the *NAEP Advanced* level should be able to defend their solutions to complex non-routine tasks. Students should be able to reason about and with functions and transformations, using properties of functions and transformations to analyze relationships and to determine and construct appropriate representations for solving problems; explain or defend reasoning processes; and understand the role of hypotheses, deductive reasoning, and conclusions in geometric proofs and algebraic arguments made by themselves and others. Students should be able to use, analyze, and justify representations created by others; use structures and patterns to generate rules and investigate the conditions under which rules apply; use a variety of grade-appropriate proof methods to justify a mathematical statement using valid definitions, statements, theorems, or counterexamples; determine and use a series of processes to mathematize a complex or non-routine situation and evaluate the results obtained; and extend, connect or generalize across the ideas of others.

NAEP Basic, NAEP Proficient, and NAEP Advanced Achievement Levels for Grade 4

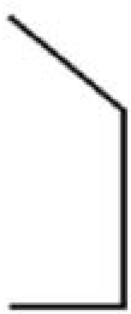
NAEP Basic, Grade 4

In this item, students are given an incomplete representation of a shape and asked to identify an associated complete shape, addressing the *NAEP Basic* level language "identify or measure attributes of simple plane figures."

Subject: Mathematics, Grade: 4, Year: 2013
Content Classifications: Geometry, Low, Type: MC, Difficulty Level: Easy

The correct answer is:

A. Pentagon



Part of a closed shape is shown above. When the shape is completed, which of these could it be?

A. Pentagon

B. Rectangle

C. Square

D. Triangle

NAEP Proficient, Grade 4

In this item, students are presented with a problem situation involving multistep computation and interpretation within the context of the situation, addressing *NAEP Proficient* level language "estimate and compute with whole numbers (within the guidelines set by the NAEP objectives)" and "abstract or de-contextualize and re-contextualize ideas in routine problems."

Subject: Mathematics, Grade: 4, Year: 2017

Content Classifications: Number properties and operations, Moderate, Type: SR, Difficulty Level: Hard

A school will receive between $600 and $900 to spend on art supplies.

The money will be given to three school clubs.

Each school club will get the same amount of money.

Which of the following amounts of money could each school club get?

Select all the correct answers.

- A ☐ $145
- B ☐ $225
- C ☐ $295
- D ☐ $325
- E ☐ $355

Clear Answer

NAEP Advanced, Grade 4

In this item, students are presented with a specific mathematical scenario and asked to generalize the results and provide a justification for the generalization, addressing *NAEP Advanced* level language "use structures and patterns to generate a rule" and "use a variety of grade-appropriate methods to justify or refute a mathematical statement [the rule] using valid definitions, statements, or counterexamples."

Subject: Mathematics, Grade: 4, Year: 2011
Content Classifications: Number properties and operations, High, Type: SCR, Difficulty Level: Hard

Mr. Jones picked a number greater than 100.

He told Gloria to divide the number by 18.

He told Edward to divide the number by 15.

Whose answer is greater?

○ Gloria's ○ Edward's

Explain how you know this person's answer will always be greater for any number that Mr. Jones picks.

NAEP Basic, NAEP Proficient, and NAEP Advanced Achievement Levels for Grade 8

For each of items 1 through 4, refer to the following three figures.

Figure 1

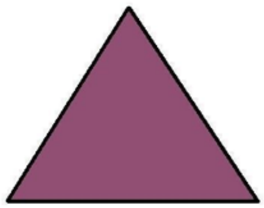


Figure 2

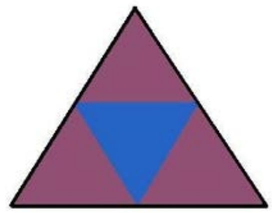
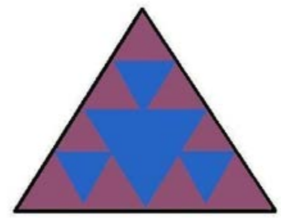


Figure 3



NAEP Basic, Grade 8

Item 1.

Figure 1 is an equilateral triangle and s is the length of a side of the triangle. P is the perimeter of the triangle in Figure 1. Complete the equation for the perimeter, P, of Figure 1. $P = \square \cdot s$

This item is an indicator of *NAEP Basic* because students are asked to recognize or apply directly procedures and representations that are routine at grade 8 regarding perimeter of triangles.

Item 2.

In Figure 2 the blue triangle has been created by connecting the midpoints of the sides of the original triangle in Figure 1. Indicate if each of the following statements is true or false: a) The perimeter of the blue triangle is one-fourth the perimeter of the original triangle   b) The perimeter of the blue triangle is one-half the perimeter of the original triangle   c) The area of the blue triangle is one-fourth the area of the original triangle   d) The area of the blue triangle is one-half the area of the original triangle

This item is an indicator of *NAEP Basic* because students are asked to recognize or apply simple relationships regarding area and perimeter of triangles.

NAEP Proficient, Grade 8

Item 3.

Figure 1 is an equilateral triangle, and s is the length of a side of the triangle. In Figure 2 the blue triangle has been created by connecting the midpoints of the sides of the original triangle. In Figure 3 the smaller blue triangles have been created by connecting the midpoints of the sides of each interior triangle in Figure 2. 1) Express the perimeter of the blue triangle in Figure 2 in terms of s. 2) Express the sum of the perimeters of all the blue triangles in Figure 3 in terms of s.

Item 3 is an indicator of *NAEP Proficient* because it involves applying a well-known procedure to solve a non-routine problem that should be accessible to grade 8 students, and representing the solution using grade-appropriate algebraic representations.

NAEP Advanced, Grade 8

Item 4.

Figure 1 is an equilateral triangle. In Figure 2 the blue triangle has been created by connecting the midpoints of the sides of the original triangle. In Figure 3 the smaller blue triangles have been created by connecting the midpoints of the sides of each interior triangle in Figure 2. Suppose you continue this process of connecting midpoints to obtain subsequent figures (Figure 4, Figure 5, Figure 6, and so on). 1) Express the sum of the perimeters of all the blue triangles in Figure 5 in terms of s. 2) Express the sum of the perimeters of all the blue triangles in Figure 10 in terms of s.

Item 4 is an indicator of *NAEP Advanced* because it involves generalizing a pattern and using a well-known procedure in the context of the pattern to solve a non-routine problem, and representing the solution using grade-appropriate algebraic representations.

NAEP Basic, NAEP Proficient, and NAEP Advanced Achievement Levels for Grade 12

NAEP Basic, Grade 12

In this item, students are given pairs of shapes and asked to identify the pair that must always be similar, addressing *NAEP Basic* level language "identify and describe relationships of congruence, similarity, and symmetry."

Subject: Mathematics, Grade: 12, Year: 2005
Content Classifications: Geometry, Low, Type: MC, Difficulty Level: Medium

The correct answer is:

A. Two equilateral triangles

Which of the following pairs of geometric figures must be similar to each other?

A. Two equilateral triangles

B. Two isosceles triangles

C. Two right triangles

D. Two rectangles

E. Two parallelograms

NAEP Proficient, Grade 12

In this item, students are asked to select the data collection method most appropriate for the question of interest, addressing *NAEP Proficient* level language "They should be able to design and carry out statistical surveys."

Subject: Mathematics, Grade: 12, Year: 2009
Content Classifications: Data analysis, Statistics, and Probability, Moderate, Type: MC, Difficulty Level: Medium

The correct answer is:

C. Randomly select 25 students from a list of all students at the school.

The principal of a high school would like to determine why there has been a large decline during the year in the number of students who buy food in the school's cafeteria. To do this, 25 students from the school will be surveyed. Which method would be the most appropriate for selecting the 25 students to participate in the survey?

A. Randomly select 25 students from the senior class.

B. Randomly select 25 students from those taking physics.

C. Randomly select 25 students from a list of all students at the school.

D. Randomly select 25 students from a list of students who eat in the cafeteria.

E. Give the survey to the first 25 students to arrive at school in the morning.

NAEP Advanced, Grade 12

In this item, students need to use geometric properties, definitions, and principles to describe a geometric process for finding the center of any circle, addressing *NAEP Advanced* level language "use a variety of grade-appropriate proof methods to justify a mathematical statement using valid definitions, statements, theorems, or counterexamples."

Subject: Mathematics, Grade: 12, Year: 1996
Content Classifications: Geometry, Problem solving, Type: ECR, Difficulty Level: Hard

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important to show all your work.

Describe a procedure for locating the point that is the center of a circular paper disk. Use geometric definitions, properties, or principles to explain why your procedure is correct. Use the disk provided to help you formulate your procedure. You may write on it or fold it any way that you find helpful, but it will not be collected.

MATHEMATICS

1. EXPANSION OF ATTENTION TO STUDENT REASONING AND MATHEMATICAL PRACTICES

We recommend defining mathematical practice constructs of priority interest in the framework (e.g., representing, abstracting and generalizing, justifying and proving, modeling, mathematical collaboration), providing examples of how they can be assessed (e.g., in the Assessment and Item Specifications), and using these definitions to systematically assess these practices, integrated with content, in 2026.

2. SIGNIFICANT BROADENING OF MATHEMATICAL DOMAINS AND COMPETENCIES

The mathematics content of the preK–12 curriculum has significantly evolved, and these changes need to be reflected in NAEP. **We recommend** a broadening of the content in several ways, including:

- (a) content that reflects research on mathematics teaching and learning that responds to students' diverse experiences, backgrounds, language, and culture;
- (b) a re-examination of statistics, data analysis and probability concepts and skills in light of current scholarship and standards documents;
- (c) attention to a wider range of technological tools available for students;
- (d) highlighting foundational mathematical themes that cut across different areas of content domains (e.g., geometry, algebra) and the grade bands from grades 4 to 8 to 12; and
- (e) consideration of a new cross-cutting theme or content area (at grade 12) that expands on calculus-readiness and statistics to include increasingly relevant applied mathematics important to informed citizenship, to personal financial and other decisions, and a variety of careers.

3. ATTENTION TO THE BALANCE OF COGNITIVE DEMAND

NAEP's current levels of "mathematical complexity" afford a balance between low-level items that ask for recall or demonstration of procedures, medium-level items that require connection-making on multistep procedures, and high-level items that require analysis, creativity, synthesis, or justification and proof. **We recommend** a NAEP mathematics framework update in terms of relevant research on mathematical complexity and cognitive demand.

TEST DESIGN AND TECHNOLOGY

4. TEST DESIGN

We recommend the integration of content and practice skills through leveraging interactive multimedia scenario-based tasks as a way to provide more authentic tasks for students to complete (e.g., NAEP Technology and Engineering Literacy; see online TEL tasks).

5. STRATEGIC USE OF TECHNOLOGY
We recommend that NAEP revisions leverage technology to increase the assessment's authenticity (allowing students to use the technologies they use in and out of school) and the assessment's accessibility. Given the digital divide, as the NAEP instrument evolves, panels should address known and potential implementation issues and recommend ways to mitigate issues of access and test-taking that could occur in under-resourced communities.

OPPORTUNITIES TO LEARN AND OPPORTUNITIES TO DEMONSTRATE LEARNING

6. EXPANSIVE CONCEPTION OF OPPORTUNITIES TO LEARN
We recommend developing a broad approach to the framework update that scaffolds attention to opportunities to learn mathematics content, processes, and practices. This intent should be woven into the objectives in the framework, the item types and examples, and realized in contextual variables used on surveys.

We recommend updates to contextual variables in surveys that include attention to students' views of mathematics, and of themselves as mathematics learners; students' views of their peers', teachers', and school's beliefs/interest in their progress in mathematics; students' views of mathematics teaching and mathematics assessment (including NAEP); student access to and engagement with the language and culture of the test; teachers' knowledge of what has been taught before NAEP is administered; and teachers' beliefs about mathematics, mathematics teaching, and what their students can do.

7. ACCESSIBLE ASSESSMENTS FOR ALL STUDENTS
We recommend developing authentic assessment items with multiple access points that provide diverse populations of students with opportunities to demonstrate their mathematical knowing and reasoning in creative, authentic ways. This includes improving the accessibility of the assessment through short term goals like reconsidering test time limits, establish testing conditions that are more closely aligned with learning conditions (the use of typical tools, for example, or allowing teachers to be present) as well as longer term efforts to document how the current assessment remains inaccessible. Items should have consequential validity, be engaging to students, reflect guidelines for "low floor, high ceiling" tasks that provide opportunities for multiple approaches, and connect to students' lived experiences and funds of knowledge. Making the testing technologies widely available to students and teachers well before the assessment would also increase access and authenticity. Finally, because some research suggests that using mathematics tasks situated in everyday situations allows students to bring greater meaning to those tasks, we believe the authenticity of assessment items may allow for a more successful assessment of the mathematics students are learning (Boaler, 2002; Tomaz & David, 2015).

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